GdR ISIS - Géometrie et représentation de la couleur

Interpolation of the MacAdam ellipses

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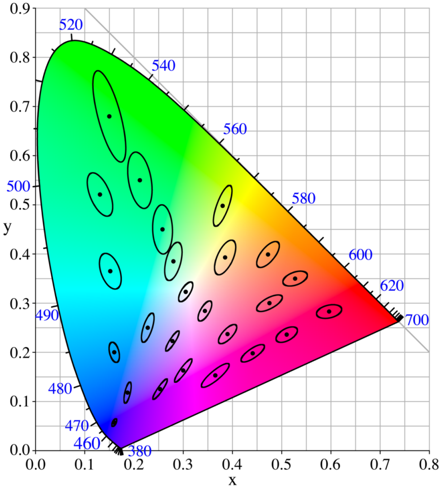
MacAdam ellipses

Sets of indistinguishable colors at $48cd/m^2$:



Riemannian geometry

- The scalar product depends on the point :

$$(\mathbb{R}^2, \langle ., . \rangle) \text{ becomes } (\mathbb{R}^2, \langle ., . \rangle_x),$$

and the length of a curve becomes

$$L(\gamma) = \int \sqrt{\langle \gamma'(t), \gamma'(t) \rangle_{\gamma(t)}} dt$$

$\langle ., . \rangle_x$: local Sym_+ matrix M_x, local ellipse $\mathcal{E}_x$

- Distance perceived by the eye :

$$\text{Riemannian hypothesis} \Rightarrow \langle ., . \rangle_{c_i} = \text{MacAdam ellipse } \mathcal{E}_i \text{ centered at } c_i$$

Parametrization change :

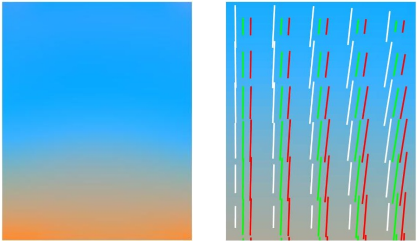
$$\varphi : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$M_x = d\varphi^T M_x^{\varphi} d\varphi$$

Interpolation of the ellipses

Goal : Determining the Riemannian geometry of colors

Challenge : How do we interpolate the MacAdam ellipses ?

- Applications :
  - image processing algorithms
  - Schrodinger conjecture : geodesic passing through grey have a constant hue



(a) (b)

FIGURE – (a) is a constant luminance image (b) shows the greatest variations according to different geometries of colors

- Multiple ways of performing the interpolation, which is the best ?

Different interpolation framework

- Parametrization of the $\mathcal{CM}$:

xy,ab and uv

- Representation of the scalar product :

- matrix of the bilinear form $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$
- M^{-1}
- $\log(M)$ (approximation of the affine-invariant metric on Sym_+)
- ellipse parameters (a, b, θ)

$$f^{(1)} : xy \longrightarrow \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 2\pi[$$

$$f^{(2)} : ab \longrightarrow \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 2\pi[$$

Which is the best framework (1) or (2) ?

Evaluation of an interpolation

For an interpolation $f^{(1)} : xy \rightarrow \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 2\pi[$ its quality could be measured by its regularity :

$$Q_1 = \int_{xy} \|df^{(1)}_{x,y}\| dxdy$$

Same interpolation in ab, $f^{(2)} : ab \rightarrow \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 2\pi[$ the measure the quality would become :

$$Q_2 = \int_{ab} \|df^{(2)}_{a,b}\| dadb$$

Problem : Q_1 and Q_2 are not comparable

Challenge : defining a criterion which is independent of a parametrization

Intrinsic evaluation

Challenge : define a criterion which is independent of a parametrization

Solutions

- the average Riemannian curvature of the inteprolation
- a cross validation approach

$$Q = \sum_{i=1}^{25} d_{\mathcal{E}_i}(\hat{\mathcal{E}}_i^{(24)})$$

where

$$d_{\mathcal{E}_{ref}}(\mathcal{E}) = \|M_{ref}^{-1/2} M M_{ref}^{-1/2} - I\|$$

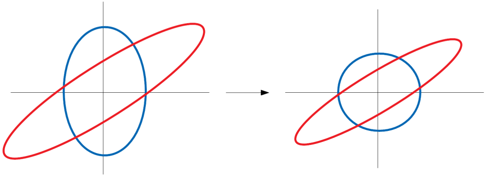


FIGURE – Blue : true ellipse, Red : interpolated ellipse

Evaluation results

TABLE – Interpolation type

L	linear
K	kernel

M	matrix
P	ellipse parameters
I	inverse

TABLE – Evaluation of the different interpolation rules

	xyY	*Lab*	*Luv*
LM	1.814	1.024	0.971
LMI	0.624	0.738	0.644
KM	1.514	1.121	0.809
KMI	0.776	0.795	0.602

	xyY	*Lab*	*Luv*
LP	0.62	0.799	0.79
LPI	0.956	0.931	0.87
KP	1.004	0.911	0.721
KPI	1.242	1.006	0.799

- Interpolations on M^{-1} are always better than on M
- Interpolations on (a, b, θ) are always better than on $(\frac{1}{a}, \frac{1}{b}, \theta)$
- *uv* gives the best results most of the time

Error maps

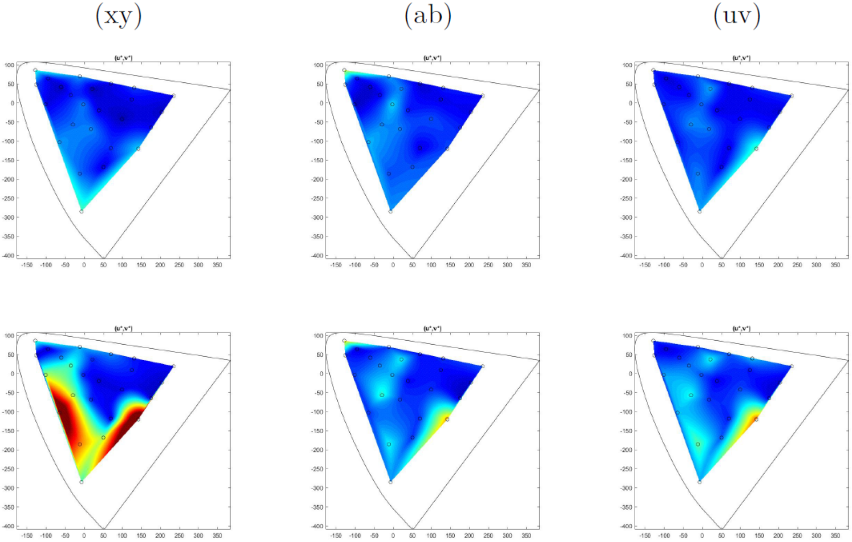
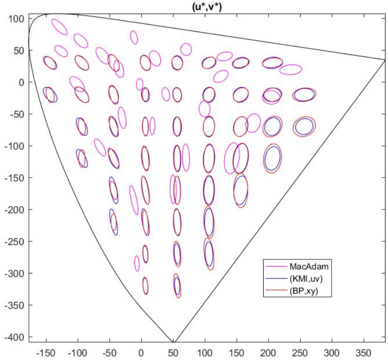
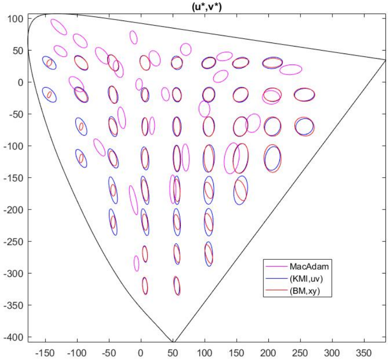


FIGURE – first row : LMI, seconde row : LM

Interpolations



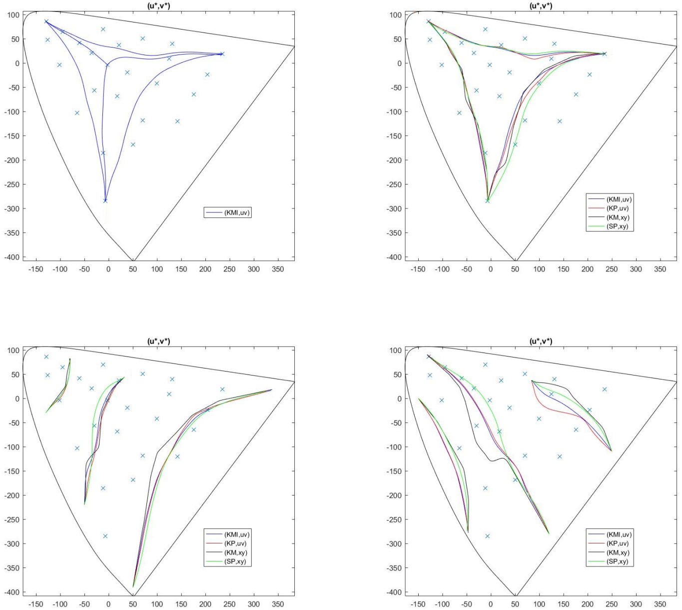
(a)



(b)

FIGURE – Plots of ellipses in *uv* coordinates, (a) : 2 best interpolations, (b) : best and worst interpolations

Geodesics


(u*,v*)
(KMI,uv)
(u*,v*)
(KMI,uv)
(KP,uv)
(KM,xy)
(SP,xy)
(u*,v*)
(KMI,uv)
(KP,uv)
(KM,xy)
(SP,xy)
(u*,v*)
(KMI,uv)
(KP,uv)
(KM,xy)
(SP,xy)
100 50 0 -50 -100 -150 -200 -250 -300 -350 -400
-150 -100 -50 0 50 100 150 200 250 300 350

Conclusion

Conclusions :

- inverse matrix of the bilinear form
- results on geodesics need further works (Schrodinger conjecture)

Perspectives :

- couple the leave one criterion out with curvature
- rephrase the problem as an optimization on the manifold of the Riemannian metrics embedded with an appropriate metric