A Neural Field model for Color Perception in context

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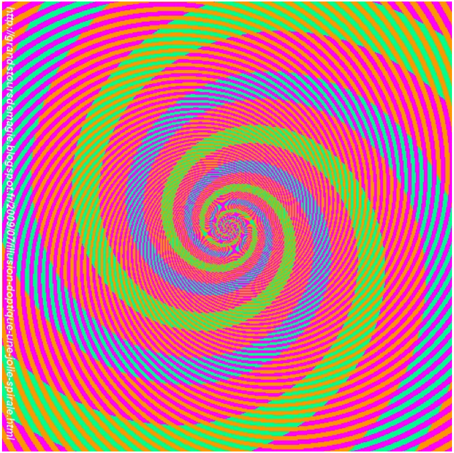
1. Introduction

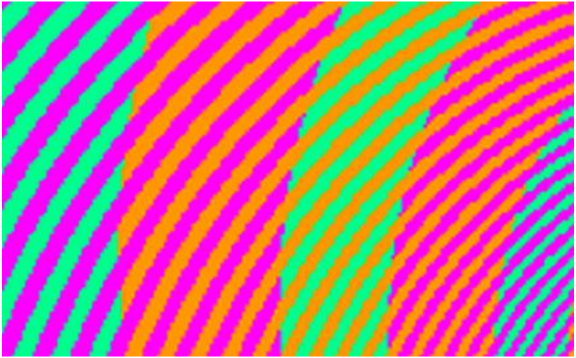
A. Color in context: some illusions

Illusion 1



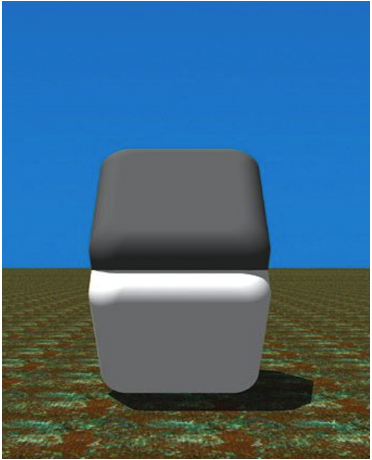
How many different colors?

Illusion 1



Only 3.

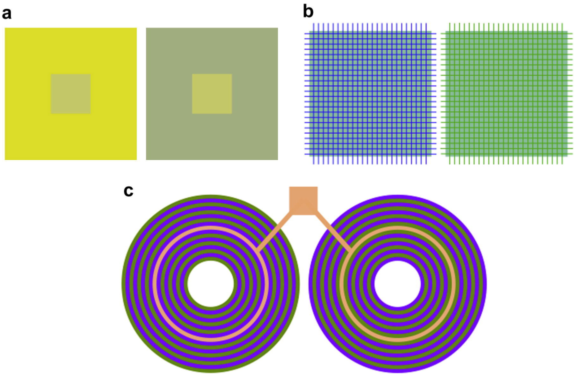
Illusion 2



Up and down: same or different?

1. Introduction

B. Assimilation and contrast



From Monnier and Shevell (2008).

a) **Simultaneous contrast.** Neighbors repell towards the opponent color.

b) **Chromatic assimilation.** Neighbors attract towards their color.

c) **Synergy** of the two effects.

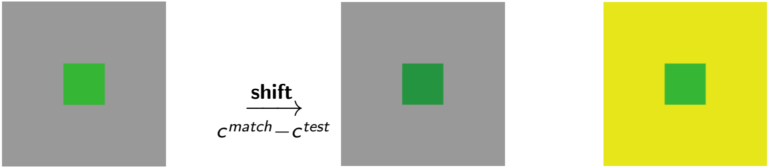
→ from global to local effects: change of vocabulary.

1. Introduction

C. Color matching experiments

Color matching

Psychophysics: How to assess perception (subjective notion) through measures (objective quantities)?



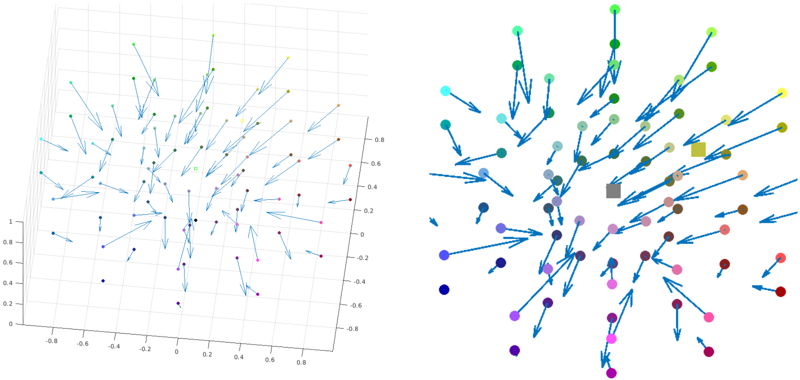
change $I^{comp}[c^{test}]$ final $I^{comp}[c^{match}]$ fix I^{test}

Among all comparison images $\{I^{comp}[c]\}_{c \in \mathfrak{C}}$ choose c^{match} which gives a **perceptual match**.

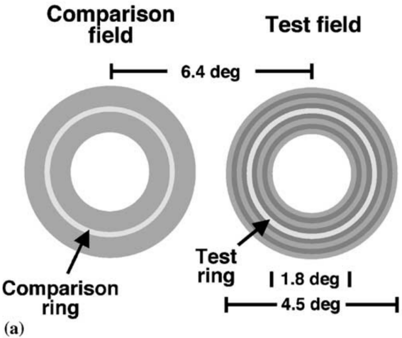
Shift $=$ difference between final and original values.

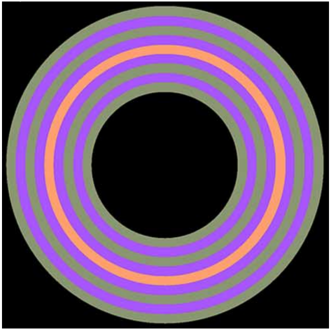
Same yellow and grey. Initialize with several colors surrounded by grey.

Results after changing appropriately the comparison images (with grey background).

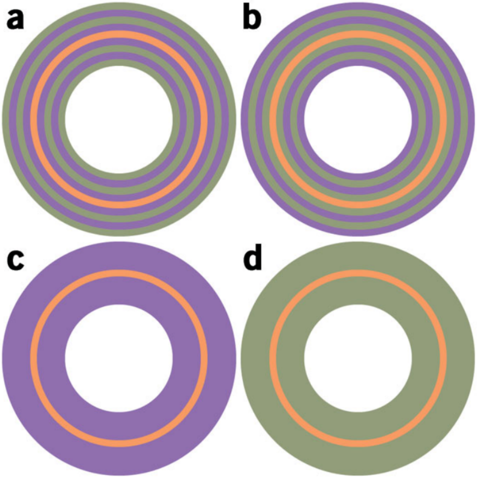


Repeat this many times: **different centers, same surrounds** $\Rightarrow$ vector field of shifts (c^{test}, c^{match}) in HSL.
Contrast wins over assimilation: 'yellow pushes towards blue' effect.

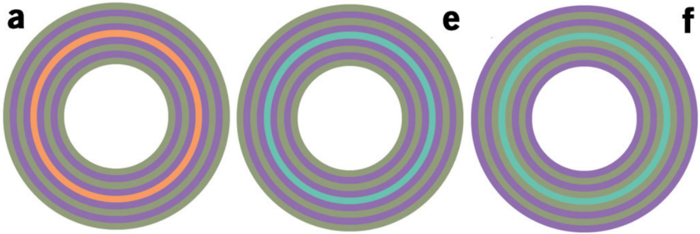




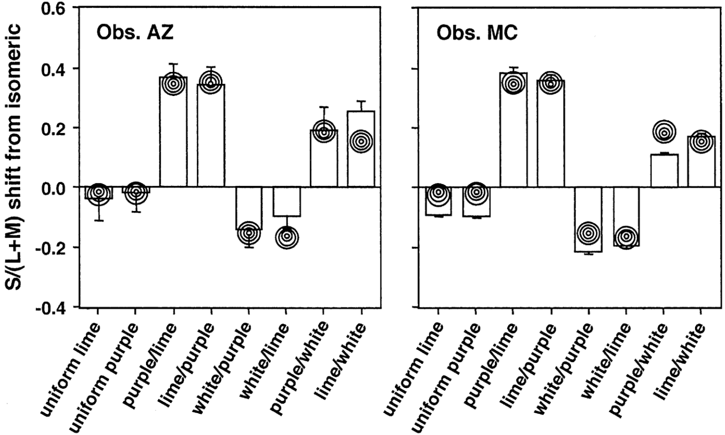
Modified from Monnier and Shevell (2004). Comparison field is a neutral background (white) with a **modifiable** comparison ring.



Modified from Monnier and Shevell (2003).
Case A: test fields with **same ring, different patterns**.



Modified from Monnier and Shevell (2003).
Case B: test fields with **different rings, same pattern**.



From Monnier and Shevell (2004).
Resulting shifts for case A, in $s := S/(L+M)$ chromaticity, where L, M, S are cones tristimulus values. Positive shifts = **towards** adjacent ring. Circles indicate their predictions.

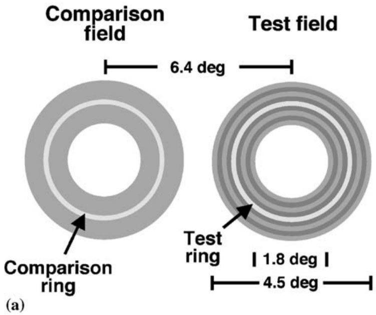
Our reformulation of their important observation:

Definition

Synergy principle.

1. *Adjacent neighbors* of point x perceptually attract towards their color.
2. *Remote neighbors* (not immediately adjacent) tend to repel towards their respective opponent color.
3. *Far neighbors* have no substantial influence on the color perception at x.

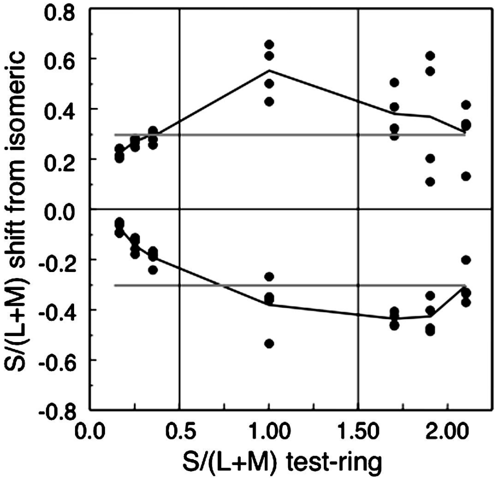
How did they make the predictions?
Synergy $\leftrightarrow$ DOG.



Model from Monnier and Shevell (2004):

$$\text{shift at } x := s^{match} - s^{test} = \text{DOG} * (I^{test} - I^{comp}[s^{test}])(x)$$

$\Rightarrow$ shift independent from s^{test} of test ring.



From Monnier and Shevell (2008). Problem occurs for **case B** (different rings, same pattern) because shifts actually **depend** on s^{test}.

Motivations of our model:

- explain the shifts from Monnier and Shevell experiments, as well as our experiments;
- synergy principle as starting point: unify assimilation and contrast at local scales;
- propose a neural field model;
- give a rigorous description of color matching experiments.

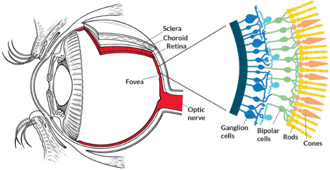
Outline

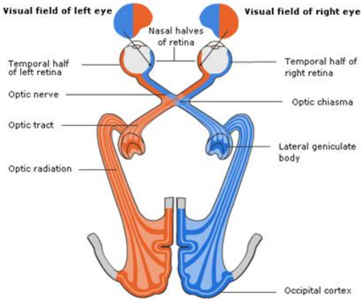
1. Introduction
   - Color in context: some illusions
   - Assimilation and Contrast
   - Color matching experiments
2. A neural field model for color perception
   - An opponent color space
   - Color Neural Field
   - Color sensation and matching experiments
3. Results and simulations
4. Discussion

2. A neural field model for color perception

A. An opponent color space

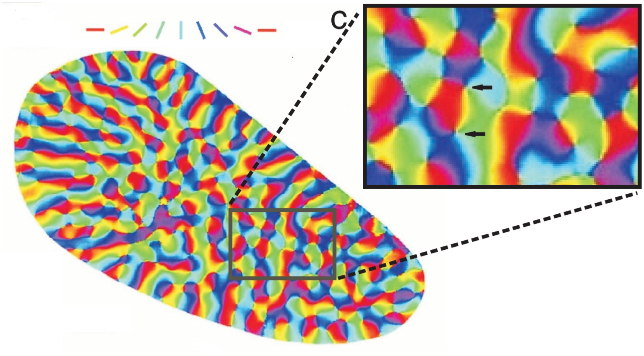
Structure of V1





Pathway of light from the eye to area V1 of the visual cortex. The retinotopic mapping is roughly one-one between the visual field and V1.

Structure of V1



Modified from Bosking et al. (1997). Cortical map of orientation selectivity.

Structure of V1

Organization into hypercolumns of orientation (neural masses sensitive to same position $r \in \Omega$, and multiple orientations). Cartesian product $\Omega \times \mathbb{S}^1$.

Assumption: similar organization into hypercolumns of colors. Cartesian product $\Omega \times \mathfrak{C}$, where $\mathfrak{C}$ is the color space that we have to define now.

Light power

Light power received by L cones at position r in the retina

$$L^r := \int_{\lambda \in \Lambda} \mathcal{P}^r(\lambda)\mathcal{R}^r(\lambda)\ \mathcal{S}_L^r(\lambda)\, d\lambda.$$

if $\mathcal{C}^r := \mathcal{P}^r\mathcal{R}^r$ denotes the spectral density emitted by the illuminant source and reflected by the object,

$$L^r = \langle \mathcal{C}^r\ ,\ \mathcal{S}_L^r \rangle_{\mathbb{L}^2(\Lambda)}$$

gives the L coordinate in (L, M, S) representation.

Metamerism

$\mathcal{C}_1, \mathcal{C}_2 \in \mathbb{L}^2(\Lambda)$ are said metameric if they produce the same visual effect.
Mathematically, $\mathcal{C}_1 \sim \mathcal{C}_2$ when they define the same triplet of scalar products

$$(L, M, S) := (\langle \mathcal{C}_i, \mathcal{S}_L \rangle_{\mathbb{L}^2(\Lambda)}, \langle \mathcal{C}_i, \mathcal{S}_M \rangle_{\mathbb{L}^2(\Lambda)}, \langle \mathcal{C}_i, \mathcal{S}_S \rangle_{\mathbb{L}^2(\Lambda)})$$

Additive structure

We define the **color vector space**

$$\mathcal{V}\mathfrak{C} := \mathbb{L}^2(\Lambda)/\sim$$

identified to $\mathbb{R}^3$ (Grassmann 1853) for trichromats through the canonical chart $\phi_{LMS} : \mathcal{V}\mathfrak{C} \to \mathbb{R}^3$.

For color blind people, $\mathcal{V}\mathfrak{C} \simeq \mathbb{R}^d$, $d \leq 2$.

Definition of the color space

The **color space** $\mathfrak{C}$ is the subspace of $\mathcal{V}\mathfrak{C}$ of lights physically realizable and visible by a human.

Property: $\mathfrak{C} \subset \mathcal{V}\mathfrak{C}$ is a positive mathematical cone (Newton 1704) and is convex (Grassmann 1853, Helmholtz 1867).

Assumption: $\mathfrak{C}$ is a bounded set instead of an infinite cone.

Representations

Many representations of $\mathfrak{C}$, through differents charts $\phi : \mathfrak{C} \to \phi(\mathfrak{C}) \subset \mathbb{R}^3$, including RGB and XYZ (linear) and canonical LMS.

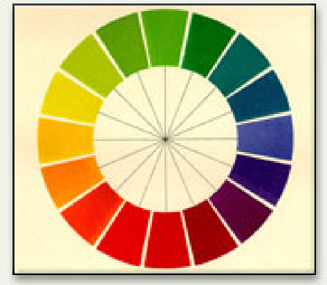
Assumption: there exists an opponent representation ϕ_{opp} of $\mathfrak{C}$ such that

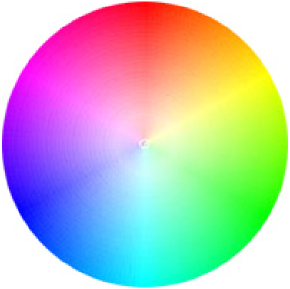
$$\mathfrak{C}_{opp} := \phi_{opp}(\mathfrak{C}) \subset \mathbb{R}^3$$

is symmetric and $c \mapsto -c$ maps c onto 'its' opponent color $-c$.

Implies the existence of **neutral color** $c = 0$.

Opponency





Left: **Hering**'s opponent color theory.
Right: HSL (Hue, Saturation,Lightness) chromatic disk.

Which representation we use here:

- **(l,s,Y) representation** (variant of MacLeod and Boynton, 1979)

$$s = S/(L+M) \quad l = L/(L+M) \quad Y = L+M$$

  $\mathfrak{C}_{opp}$ is 1D and $c := s - 1 \in \mathfrak{C}_{opp} := [-2, 2]$ (purple: $c = 1.00$, lime: $s = -0.84$).

- **HSL representation** $\mathfrak{C}_{opp}$ is 2D, the chromatic disk of constant lightness $L = 1/2$.

2. A neural field model for color perception

B. Color Neural Field

Dynamics

We suppose that the neural activity $a(r, c, t)$ evolves according to

$$\tau \frac{da}{dt} = -a(t) + F(\omega \star a(t) + H) \qquad a(t) \in \mathbb{L}^{\infty}(\Omega \times \mathfrak{C}_{opp})$$

- activation function F is a sigmoid: $F(x) := \frac{1}{1+e^{-\gamma x}}$
- cortical image $I(r, t) \in \mathfrak{C}_{opp}$ in opponent coordinates
- color input H sent by the LGN:

$$H(r, c, t) := h(c - I(r, t)), \quad h(c) := \mu_h e^{-\frac{||c||^2}{2\sigma_h^2}}$$

- typical speed of the dynamics $\tau = 1$ w.l.o.g.

$$\tau \frac{da}{dt} = -a(t) + F(\omega \star a(t) + H) \qquad a(t) \in \mathbb{L}^\infty(\Omega \times \mathfrak{C}_{opp})$$

connectivity kernel ω encodes synergy of assimilation and contrast:

$$\omega \star a(t) = \int_\Omega \int_{\mathfrak{C}_{opp}} g(r-r')f(c,c')a(r',c',t)\, dr'dc'$$

- g is a DOG:

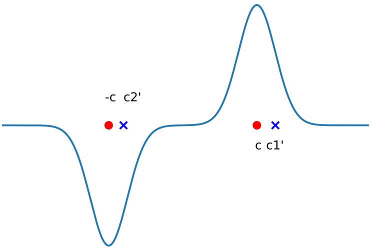
$$g(r) := \mu e^{-\frac{||r||^2}{2\alpha^2}} - \nu e^{-\frac{||r||^2}{2\beta^2}}.$$

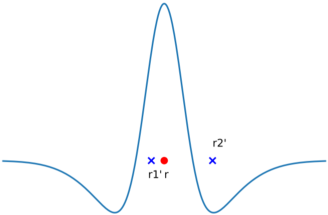
- $f(c,c')$ such that for fixed c', $f(\cdot,c')$ is a non-centered difference of gaussians

$$f(c,c') := \mu_c e^{-\frac{||c-c'||^2}{2\alpha_c^2}} - \nu_c e^{-\frac{||c+c'||^2}{2\beta_c^2}}$$

Sign of the connectivity kernel ω

$g(r-r')f(c,c')$	c' close to c	c' close to $-c$
r' close to r	>0	<0
r' far from r	<0	>0





$f(c,\cdot)$ with c fixed, and $g(r-\cdot)$ with r fixed, as functions on 1D axis.

2. A neural field model for color perception

C. Color sensation and matching experiments

We know how the cortical area reacts to an image. But how to confront the model to ground truth data?
→ notion of **color sensation** when viewing a fixed image I.
→ color matching is a mathematical projection

I is a fixed cortical image.

Definition (Color sensation)

Suppose that there exists a unique stationary solution to which the dynamics converges, denoted $a_I(\cdot,\cdot,\infty) := \lim_{t\to\infty} a_I(\cdot,\cdot,t)$. Then the *color sensation* perceived at a cortical point r_0 is

$$a_I(r_0,\cdot,\infty) : \mathfrak{C}_{opp} \to [0,1].$$

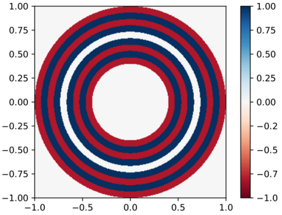
We have a test image I^{test}, a family of comparison images $\{I^{comp}[c]\}_{c \in \mathfrak{C}}$, and two reference locations r^{test} and r^{comp}. Denote a^{test} and $a^{comp}[c]$ the associated color sensations.

Proposition (Color matching is a projection)

A *color matching experiment* consists in choosing $c^{match} \in \mathfrak{C}$ so that $a^{comp}[c^{match}]$ is the *closest* to a^{test}, in the sense that c^{match} satisfies

$$c^{match} := \arg\min_{c} \; \|a^{test} - a^{comp}[c]\|_{\mathbb{L}^\infty(\mathfrak{C}_{opp})}. \tag{1}$$

3. Results and Simulations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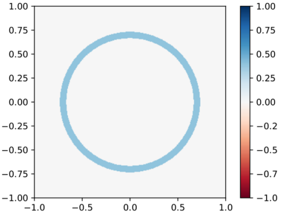
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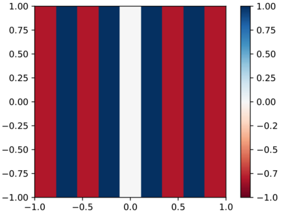
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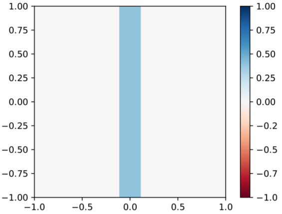
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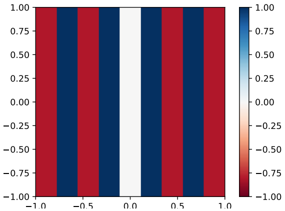
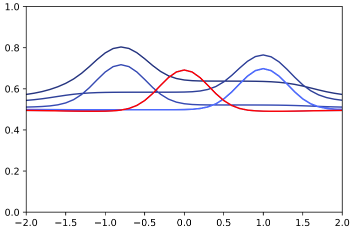
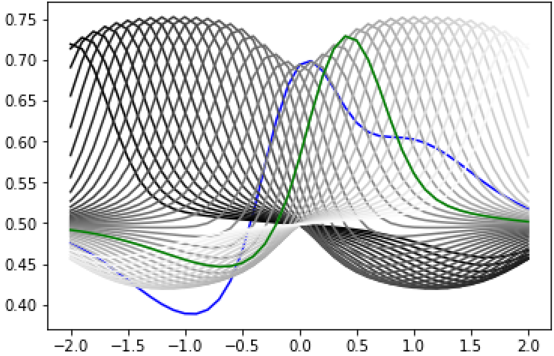
Color sensations





Matching is a projection



a^{test} in blue, $a^{comp}[c^{match}]$ in green, which is closest to a^{test} among all color sensations $a^{comp}[c]$.

Confrontation to data

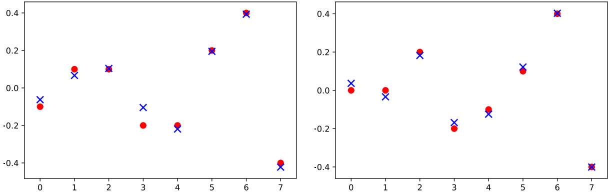
We had to regress the models (11 scalar parameters!) using **PyTorch** and a neural network structure.
Regression: minimize

$$\arg\min_q E(q) := \sum_{i=1}^{N_{exp}} (c_q^{pred}[i] - c^{match}[i])^2$$

where for each experiment $i = 1, ..., N_{exp}$, $c_q^{pred}[i]$ is the minimizer of:

$$c_q^{pred}[i] := \arg\min_c ||a_q^{test}[i] - a_q^{comp}[c]||_{\mathbb{L}^\infty(\mathfrak{C}_n)}$$

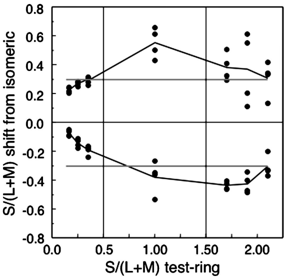
Confrontation to data of Monnier and Shevell (2004)

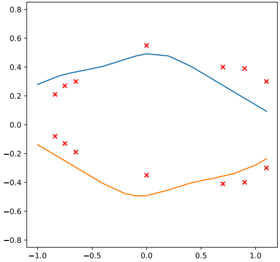


Our predictions for dataset of M&S (2004) (**case A**) corresponding to 2 observers. Red indicate ground truth. Same ring, different patterns: p/p, l/l, p/w, l/w, w/p, w/l, p/l, l/p.

Confrontation to data of Monnier and Shevell (2008)

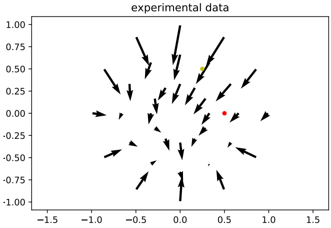
This was the initial motivation of our research.

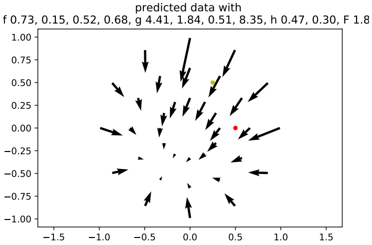




Left: dataset M&S (2008) (**case B**). Right: our predictions. Red crosses indicate ground truth. Different rings, same pattern.

Confrontation to our data (2017)





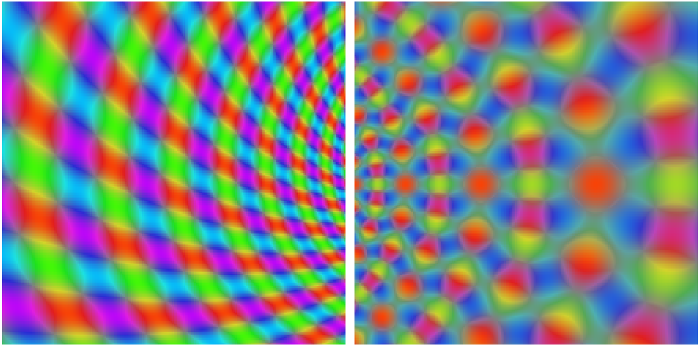
Left: our data: shifts in the HSL chromatic disk. Right: predicted results, after regression.

4. Discussion and Conclusion

- Our model is not an image processing algorithm. The problem with 'the' perceived color.
- Color matching as a projection $\rightarrow$ psychophysics; absolute *vs* relative scale
- Perfectly symmetrical opponent representation $\mathfrak{C}_{opp}$?
- The case of luminance *vs* chromaticity?
- The role of edges?

Towards color hallucinations

Equivariant bifurcation analysis should lead to color hallucinations.



(Preliminary results)

Conclusion

- First neural field model for color perception in context. However hypercolumnar structure to be biologically proven.
- Justifies some nonlinear behavior observed in color shifts.
- Color matching as a projection.
- ‘Color sensation’ instead of ‘the’ perceived color $\rightarrow$ applications to other perceptions?

[0] **A. Song, O. Faugeras, R. Veltz, A neural field model for color perception unifying assimilation and contrast, arxiv (2018)**.
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