ALGEBRA FOR COLOR CORRECTION IN IMAGE SENSORS

GENERAL CONTEXT

- **PhD student at CEA-LETI (Grenoble) with LPNC:**
  - Supervisors:
    - Jérôme Vaillant (CEA)
    - David Alleysson (LPNC)
  - Tittle: color restitution in low light level image sensors
  - Main deals:
    - Poor signal to noise ratio
    - Illumination conditions

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Classical RGB | Low light sensor

Poor contrast | False colors

PLAN

- **Introduction and context**
  - The goal of color sensor design
  - Classical color correction (state of the art)
- **Physics of light capture and sensor limitation**
  - Photon absorption, noise
- **Mathematical representation of light**
  - Discrete approximation
  - Sensor space, color space
  - Color correction interpretation
- **Challenge and prospects**

INTRODUCTION AND CONTEXT

- **Color image sensor:**
  - Goal: acquire similar data as the human perception in given conditions:

Color sensor

Human visual system

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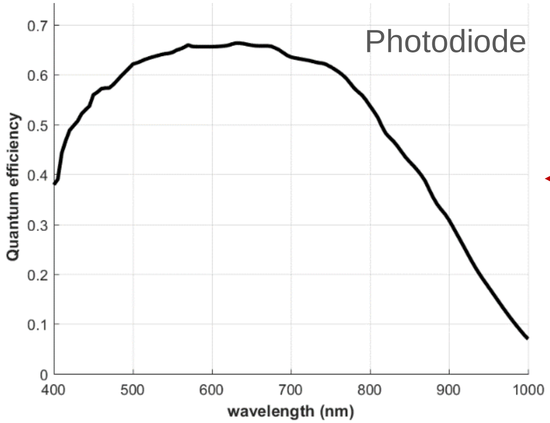
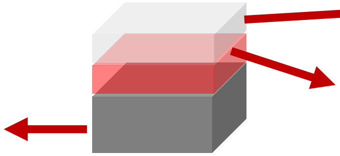
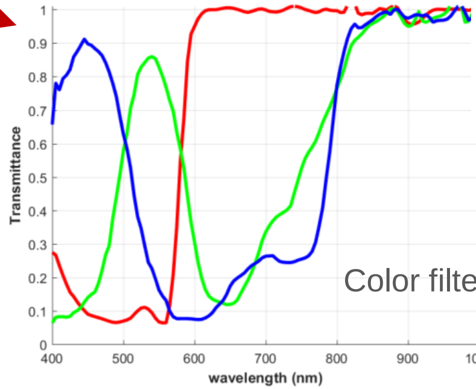
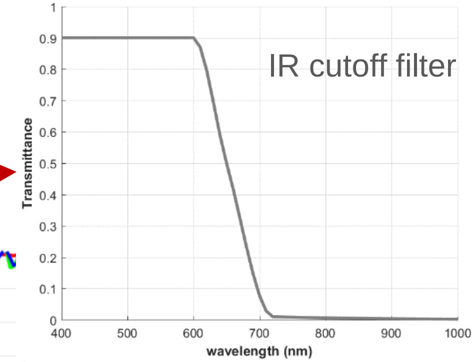
  - Key components for color acquisition (recall):





Pixel





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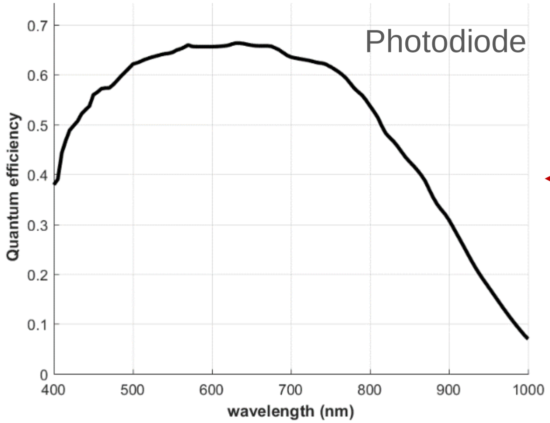

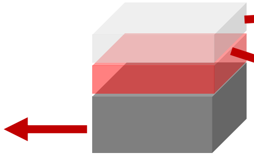

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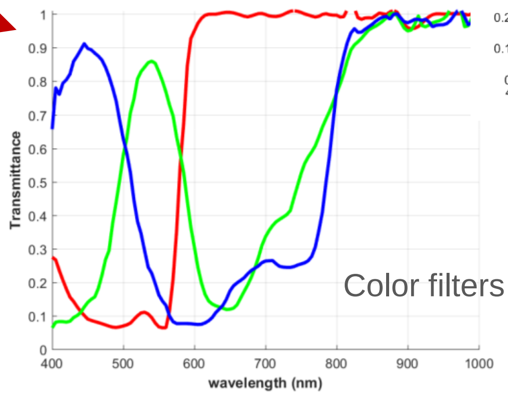
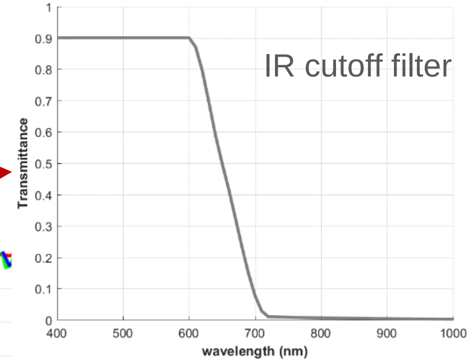


Color filters



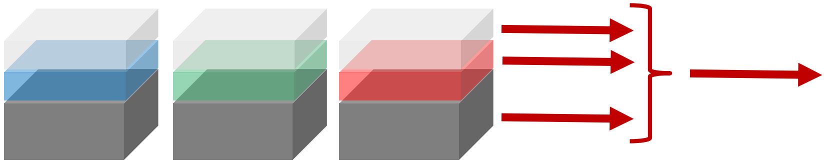
Optimisation

- **Color image sensor:**
  - Goal: acquire similar data as the human perception in given conditions:

Color sensor

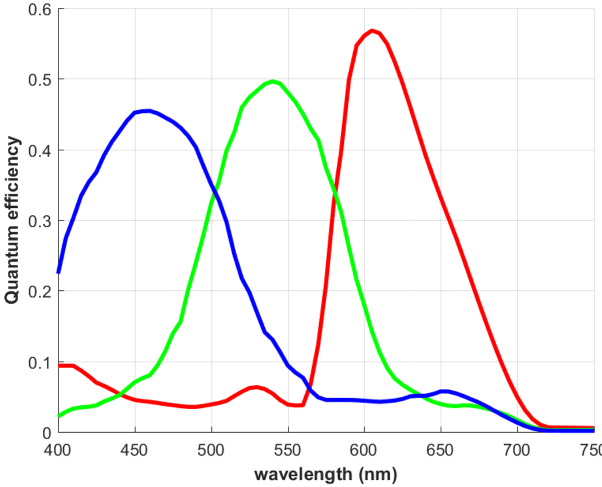
Human visual system

  - Key components for color acquisition (recall):



Pixels

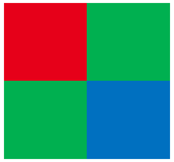
Usual « spectral sensitivities »



INTRODUCTION AND CONTEXT

- **State of the art:**
  - Basic image processing:

Demosaiçing



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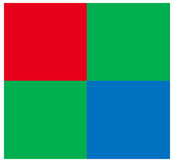
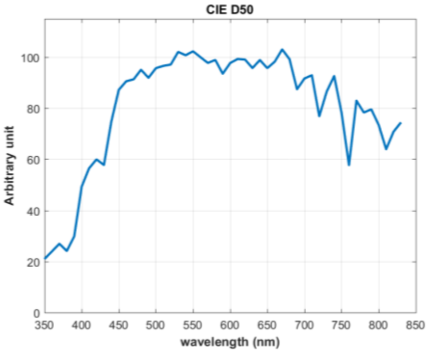
White balance





Illuminant

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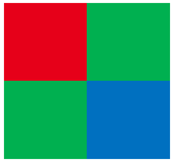
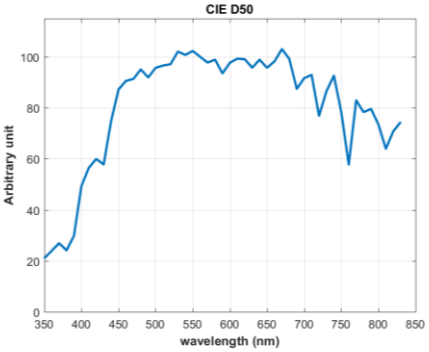
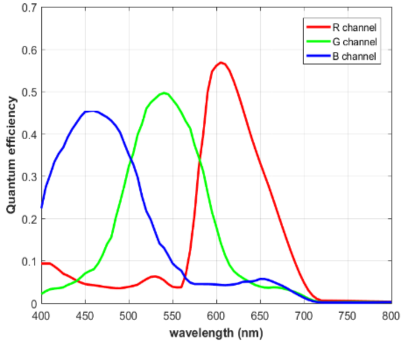
Color correction

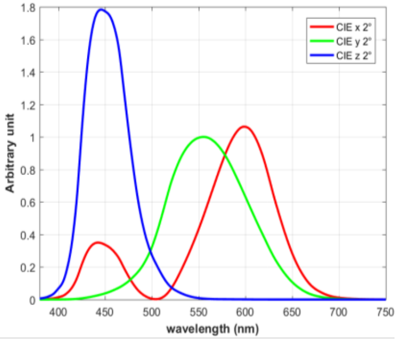




Illuminant



QE

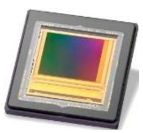


Color matching functions

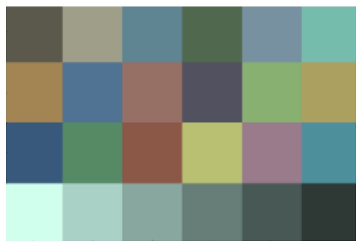
INTRODUCTION AND CONTEXT

- **State of the art:**
  - Classical color correction:

$$T = \begin{bmatrix} R_{T1} \dots R_{T24} \\ G_{T1} \dots G_{T24} \\ B_{T1} \dots B_{T24} \\ 1 \dots 1 \end{bmatrix}$$



$$S = \begin{bmatrix} R_{S1} \dots R_{S24} \\ G_{S1} \dots G_{S24} \\ B_{S1} \dots B_{S24} \\ 1 \dots 1 \end{bmatrix}$$



$$\widehat{M} = argmin_M(\|T - M.S\|^2)$$

$$\widehat{M} = T.S^T.(S.S^T)^{-1}$$

$$\widehat{M} = \begin{bmatrix} K & [V] \\ 0 \dots 0 & 1 \end{bmatrix}$$

$$K = CCM.WB$$

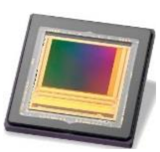
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{offset} = -K^T.\left(K.K^T\right)^{-1}.V$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{corr} = \underbrace{\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}}_{\text{CCM}} . \underbrace{\begin{pmatrix} W_{11} & 0 & 0 \\ 0 & W_{22} & 0 \\ 0 & 0 & W_{33} \end{pmatrix}}_{\text{WB}} \left(\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{raw} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}_{offset} \right)$$

INTRODUCTION AND CONTEXT

- **State of the art:**
  - Exemple of color correction (computed on hyperspectral image)
  - CIE-D65 illuminant
  - Teledyne Onyx sensor with infrared cutoff filter (slides before)

Color correction



Raw data

Corrected sRGB data

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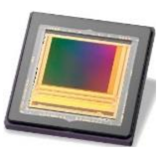


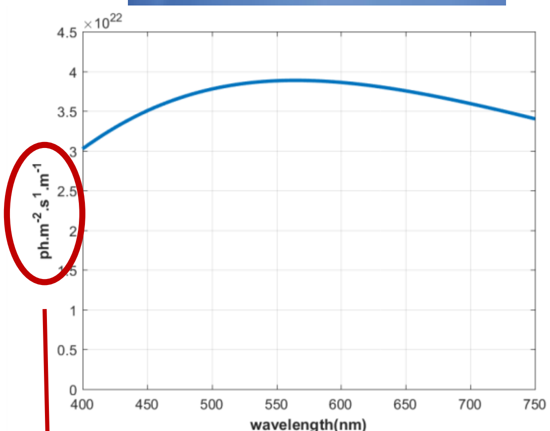
Raw data

Corrected sRGB data

Reference sRGB data

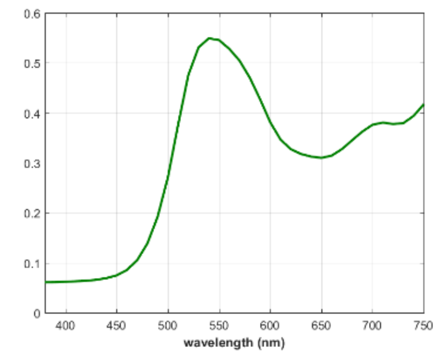
PHYSICS OF LIGHT YIELDING AND SENSOR LIMITATION

- **Photon absorption computation:**
  - Physical units:

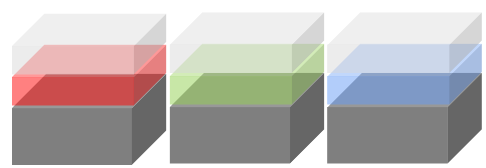


Black body 6500K

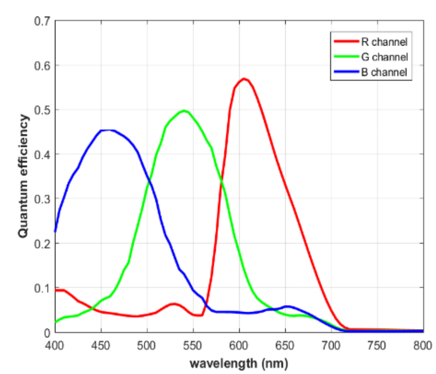
Physical absolute unit



Reflectance rate
$R_p(\lambda)$



Pixels



Photon/electron conversion rate
$QE(\lambda)$

PHYSICS OF LIGHT YIELDING AND SENSOR LIMITATION

- **Photon absorption computation:**
  - Electron measure:

$$\boxed{\mathbf{M}_{\mathbf{e\text{-}}} = \frac{N_{lux}.T_{int}.a_{pix}^2}{4.f_{\#}^2} \int_0^{\infty} I_{lum}(\lambda).\, R_p(\lambda).\, \mathrm{QE}(\lambda).\, \mathrm{d}\lambda}$$

  - Limitation of a sensor, the noise:
    - Noise: uncertainty on the electron number measurment
    - Main source of noise:
      - Readout noise (standard deviation given by the manufacturer, some electrons)
        Gaussian
      - Photonic shot noise ($\sigma_{ph}^2 = signal$)
        Poissonian

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NOISE PROPAGATION

- **Color correction:**
  - Signal and noise amplification
  - Signal to noise ratio decrease

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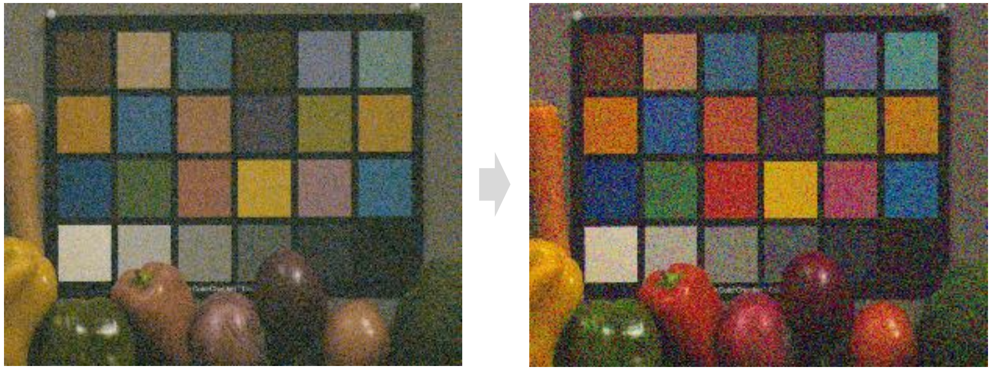
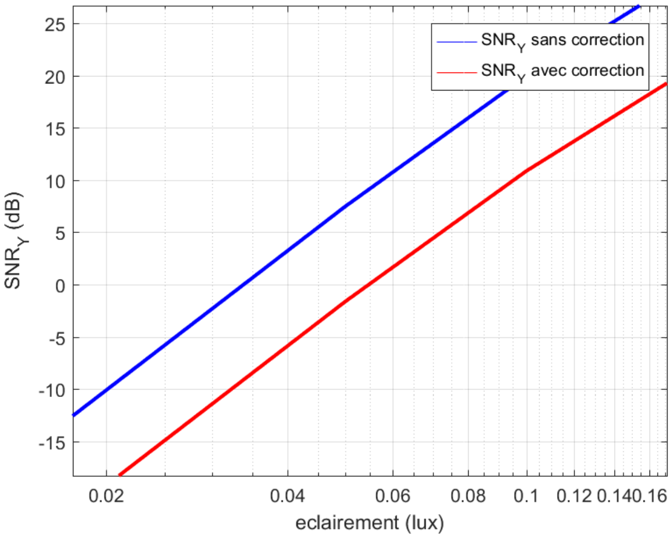


- **Quantitative exemple:**

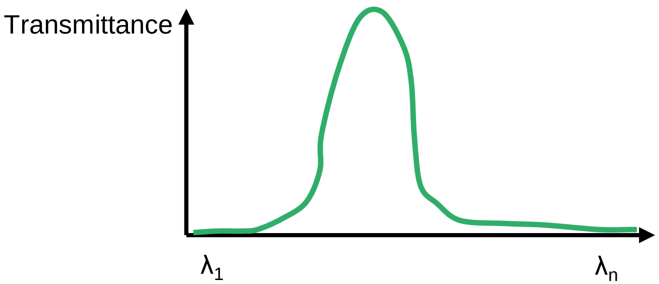
Usual model: « without correlation between channels»



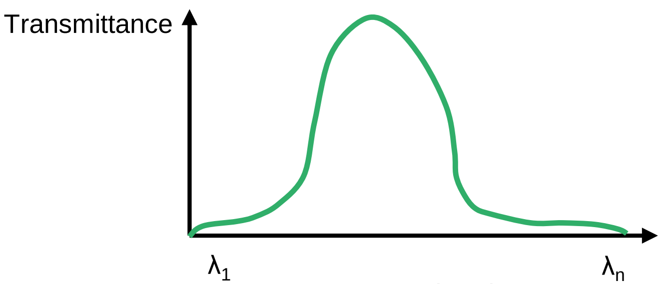
Grey patch 18%

NOISE PROPAGATION

- **Study of noise propagation → adapte spectral transmittances**
  - Use large band filter for better raw SNR:



Better spectral property



Larger band

→ Higher raw SNR

  - Final SNR increase? Not obvious
    - Noise can be amplified according to spectral shape and number of spectral channels
    - → We need an algebraic representation for noise propagation through color correction

ALGEBRAIC WRITTING

- **From physics to algebraic representation:**

$$\mathrm{M_{e\text{-}}} = \frac{N_{lux}.T_{int}.a_{pix}^2}{4.f_{\#}^2}\int_0^{\infty} I_{lum}(\lambda).R_p(\lambda).QE(\lambda).d\lambda$$

Finite support

$$\mathrm{M_{e\text{-}}} = \frac{N_{lux}.T_{int}.a_{pix}^2}{4.f_{\#}^2}\int_{400nm}^{700nm} I_{lum}(\lambda).R_p(\lambda).QE(\lambda).d\lambda$$

$$\mathrm{M_{e\text{-}}} \propto \int_{400nm}^{700nm} I_{lum}(\lambda).R_p(\lambda).QE(\lambda).d\lambda$$

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Discretisation:

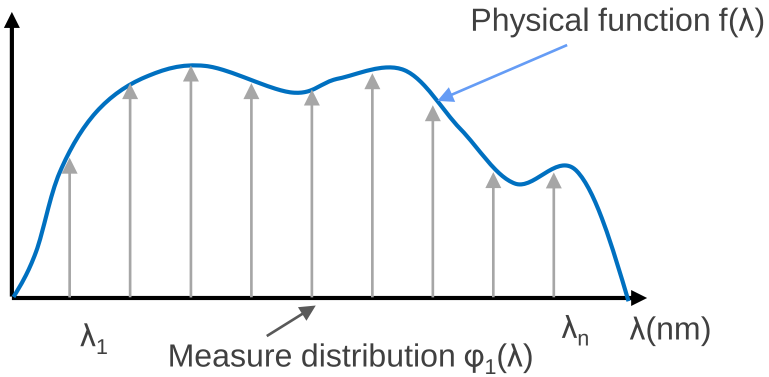
$$M_{e-} \propto \sum_{k=1}^{n} I_{lum}(\lambda_k).R_p(\lambda_k).QE(\lambda_k).\Delta\lambda$$

$$M_{e-} \propto \mathbf{QE}^T.diag(\boldsymbol{I_{lum}}).\boldsymbol{R_p}$$

Matrix form

MATHEMATICAL REPRESENTATION OF LIGHT

- **Discrete approximation of physics:**
  - Measure or digital interpolation:



Vectorized function **f**

$$\begin{bmatrix} f(\lambda_1) \\ f(\lambda_2) \\ \ldots \\ f(\lambda_n) \end{bmatrix} \quad \begin{matrix} = (f * \varphi_1)(\lambda) \\ \\ \\ = (f * \varphi_n)(\lambda) \end{matrix}$$

MATHEMATICAL REPRESENTATION OF LIGHT

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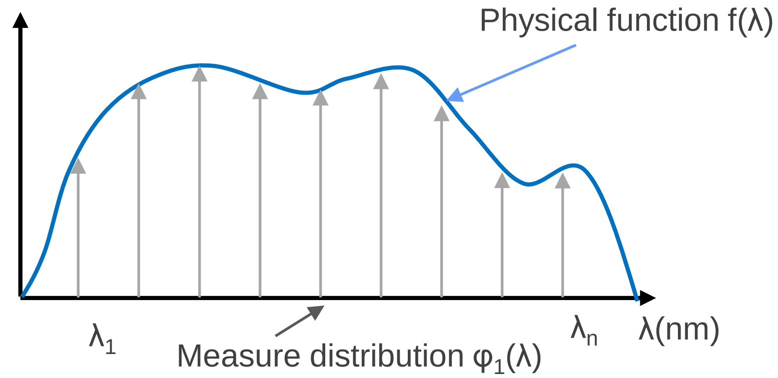


Vectorized function **f**

$f(\lambda_1)$	$= (f * \varphi_1)(\lambda)$
$f(\lambda_2)$	
...	
$f(\lambda_n)$	$= (f * \varphi_n)(\lambda)$

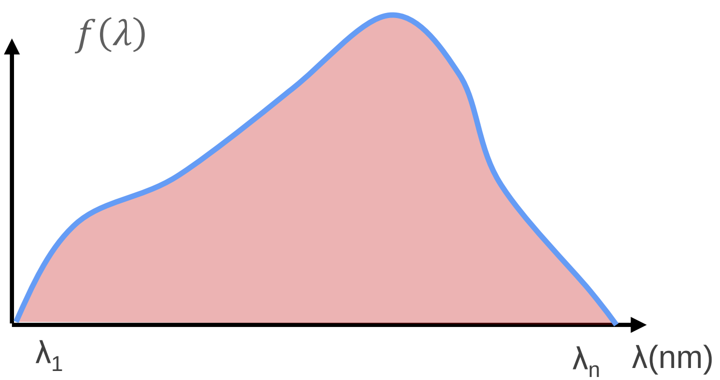
  - Vector representation:
    - n-space → R^n generated by the canonic base: a finite dimension Hilbert space
      - $\vec{f} \in R^n$
      - $\vec{f} = \sum_{k=1}^{n} f_k \,.\, \vec{e_k}$
      - $[\vec{e_1}, .., \vec{e_n}] = [(1,0, \dots, 0), \dots, (0, \dots, 0,1)]$

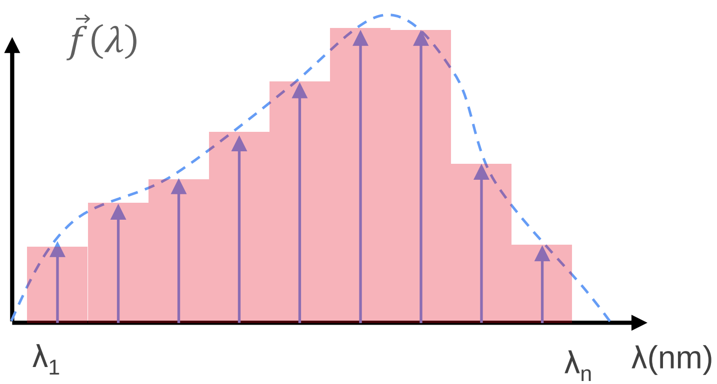
MATHEMATICAL REPRESENTATION OF LIGHT

- **Discrete approximation of physics:**
  - Formalism recalling:
    - Consider $f(\lambda)$, a spectral function
    - $\vec{f}$ is the vectorized form of f

Integration in pre-Hilbert space: « integration » in R^n:

$$\int_{\lambda_1}^{\lambda_n} f(\lambda) . d\lambda \approx \sum_{k=1}^{n} f(\lambda_k) . \Delta\lambda = [\overrightarrow{f} . \overrightarrow{1}] . \Delta\lambda$$





MATHEMATICAL REPRESENTATION OF LIGHT

- **Discrete approximation of physics:**
  - Formalism recalling:

Vectorized function **f**

$f(\lambda_1)$
$f(\lambda_2)$
…
$f(\lambda_n)$

Vectorized function **g**

$g(\lambda_1)$
$g(\lambda_2)$
…
$g(\lambda_n)$

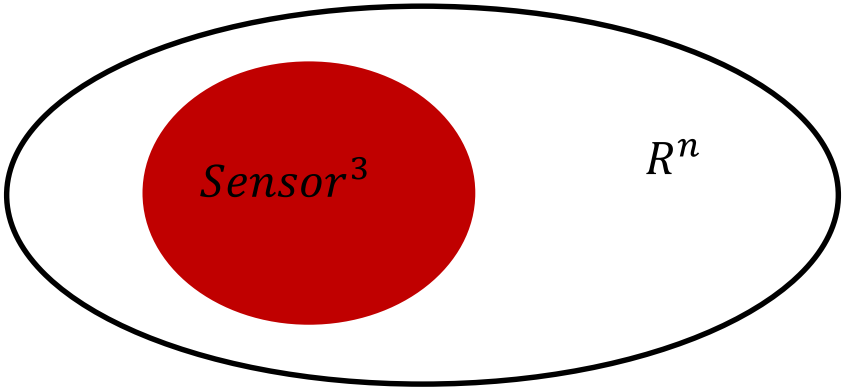
Matrix writting:

$$a = \vec{f}.\vec{g} = \boldsymbol{f}^{\boldsymbol{T}}\boldsymbol{g}$$

  - For vectorial study no need to scale the scalar products (physical parameters of $\Delta\lambda$)

MATHEMATICAL REPRESENTATION OF LIGHT

- **Sensor algebraic space:**
  - Consider an RGB image sensor having spectral channels such as $(\vec{r}, \vec{g}, \vec{b})$ are R^n elements.
    - With : $\vec{r} = \sum_{k=1}^{n} r_k \,.\, \overrightarrow{e_k}$, $\vec{g} = \sum_{k=1}^{n} g_k \,.\, \overrightarrow{e_k}$, $\vec{b} = \sum_{k=1}^{n} b_k \,.\, \overrightarrow{e_k}$
  - This family is free and generates a 3 dimensions R^n subspace:



MATHEMATICAL REPRESENTATION OF LIGHT

- **Color (or display) algebraic space:**
  - Consider a standard color space such as XYZ generated in R^n by $(\vec{x}, \vec{y}, \vec{z})$.
    - With : $\vec{x} = \sum_{k=1}^{n} x_k \,.\, \overrightarrow{e_k}$, $\vec{y} = \sum_{k=1}^{n} y_k \,.\, \overrightarrow{e_k}$, $\vec{z} = \sum_{k=1}^{n} z_k \,.\, \overrightarrow{e_k}$
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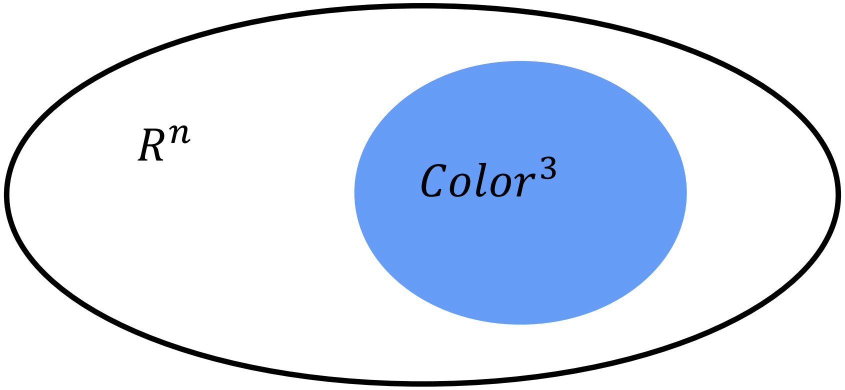


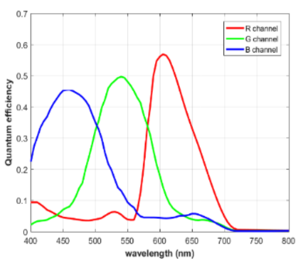
MATHEMATICAL REPRESENTATION OF LIGHT

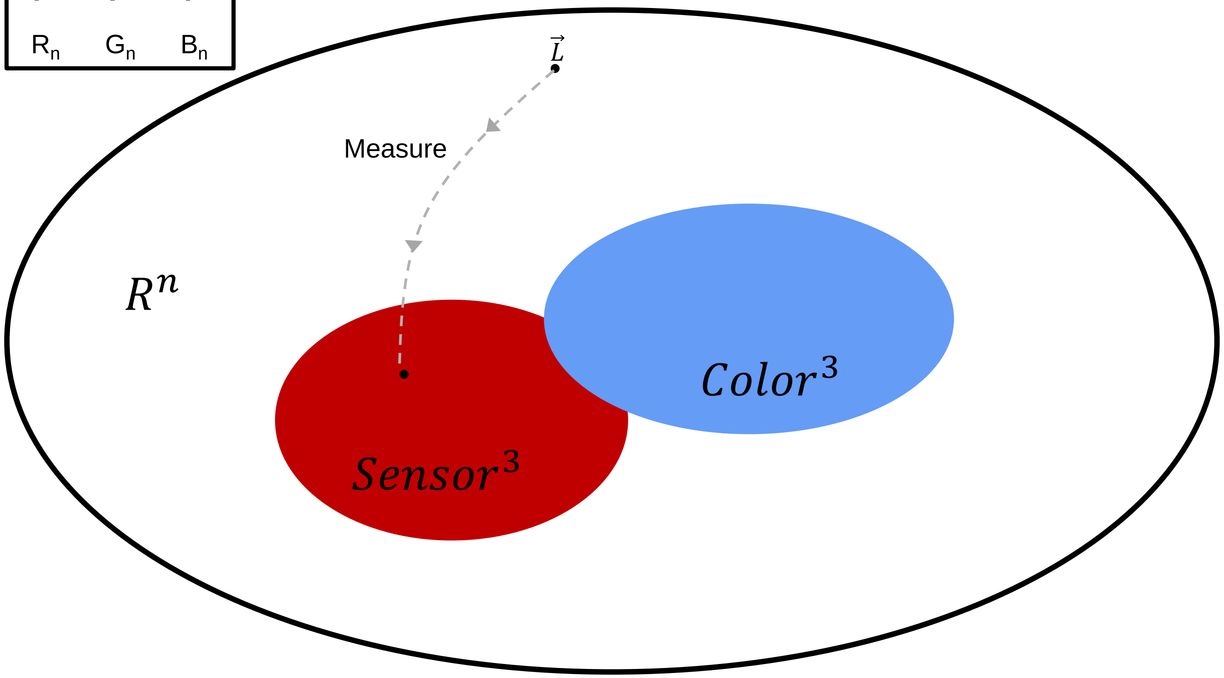
- **Schematic radiance projection:**
  - **L** radiance spectrum, **F** quantum efficiencies, **H** color matching functions

Measure operator:

$$F = \begin{bmatrix} R_1 & G_1 & B_1 \\ \vdots & \vdots & \vdots \\ R_n & G_n & B_n \end{bmatrix}$$

$$Measure = \boldsymbol{F^T}.\boldsymbol{L}$$





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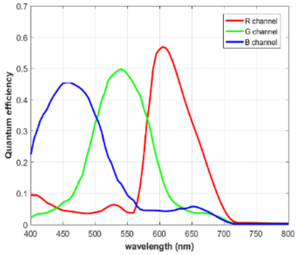
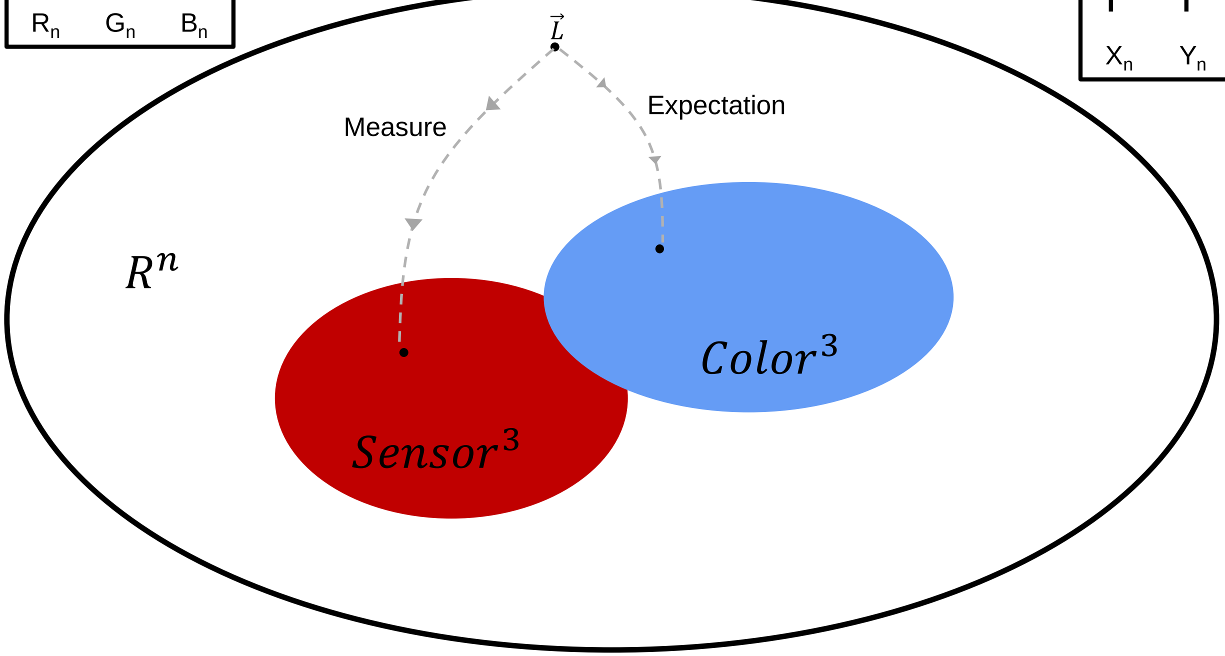
$$Measure = \boldsymbol{F^T.L}$$

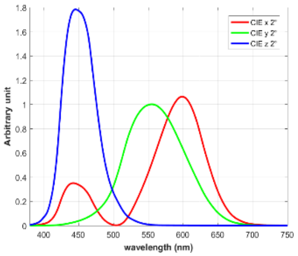
Expectation operator:

$$H = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ \vdots & \vdots & \vdots \\ X_n & Y_n & Z_n \end{bmatrix}$$

$$Expectation = \boldsymbol{H^T.L}$$







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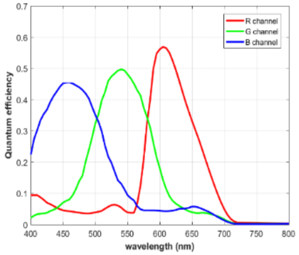
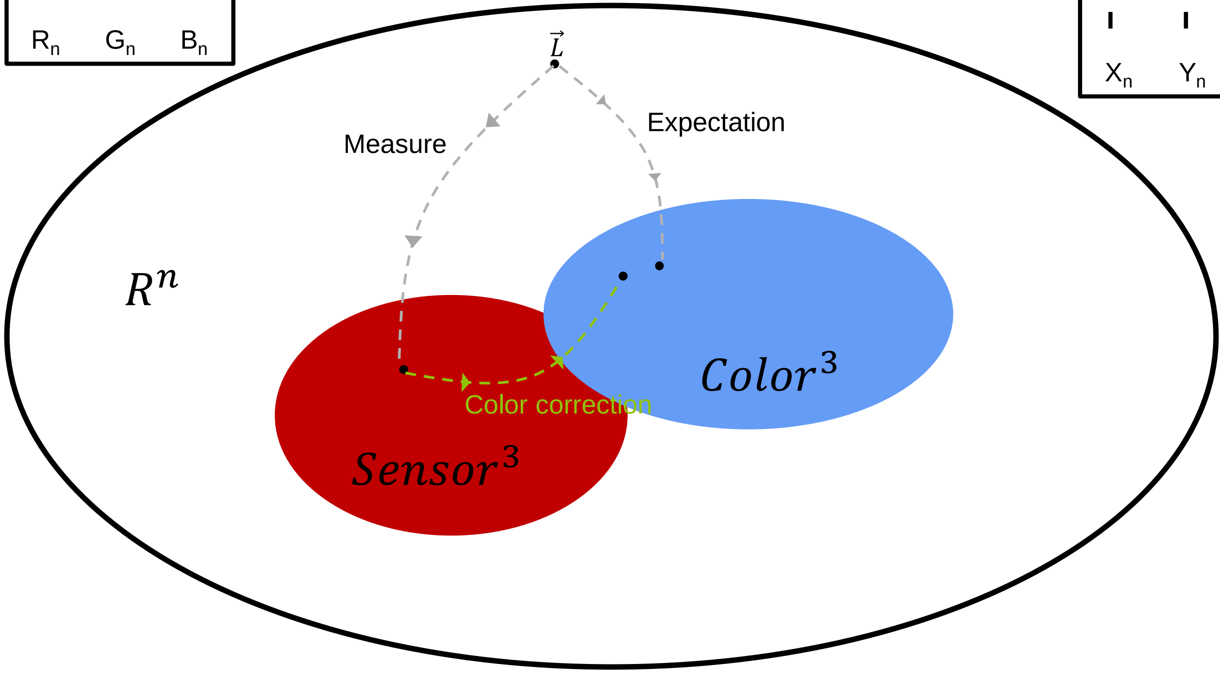
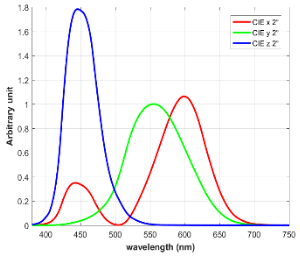
$$H=\begin{bmatrix} X_1 & Y_1 & Z_1 \\ \vdots & \vdots & \vdots \\ X_n & Y_n & Z_n \end{bmatrix}$$

$$Expectation = \boldsymbol{H^T}.\boldsymbol{L}$$

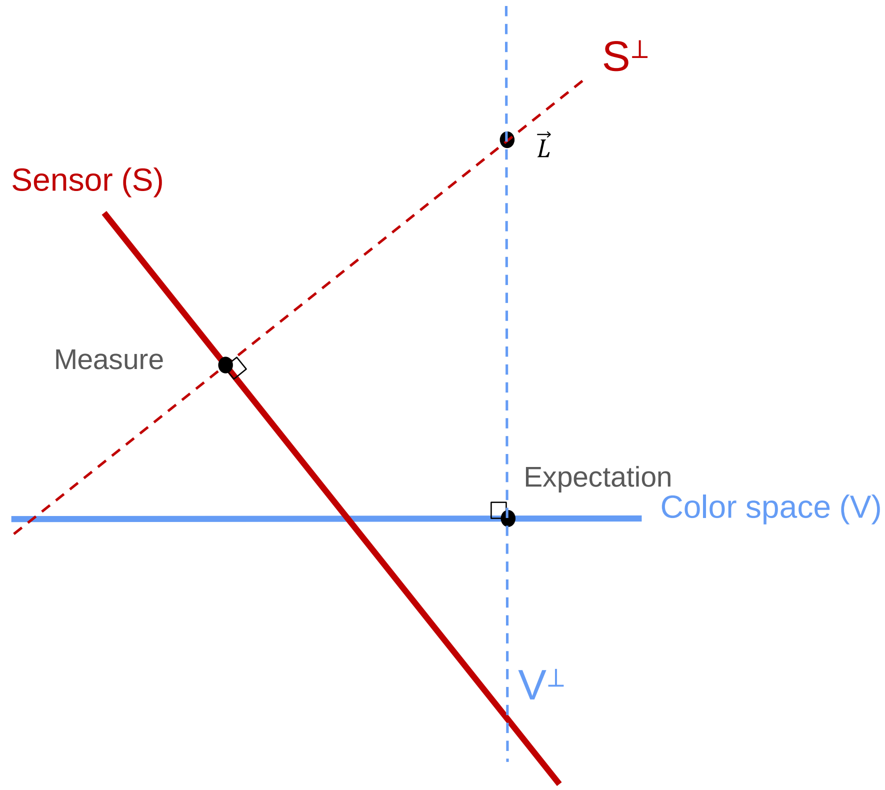






MATHEMATICAL REPRESENTATION OF LIGHT

- **Another schematic view of color correction:**
  - Raw acquisition:



MATHEMATICAL REPRESENTATION OF LIGHT

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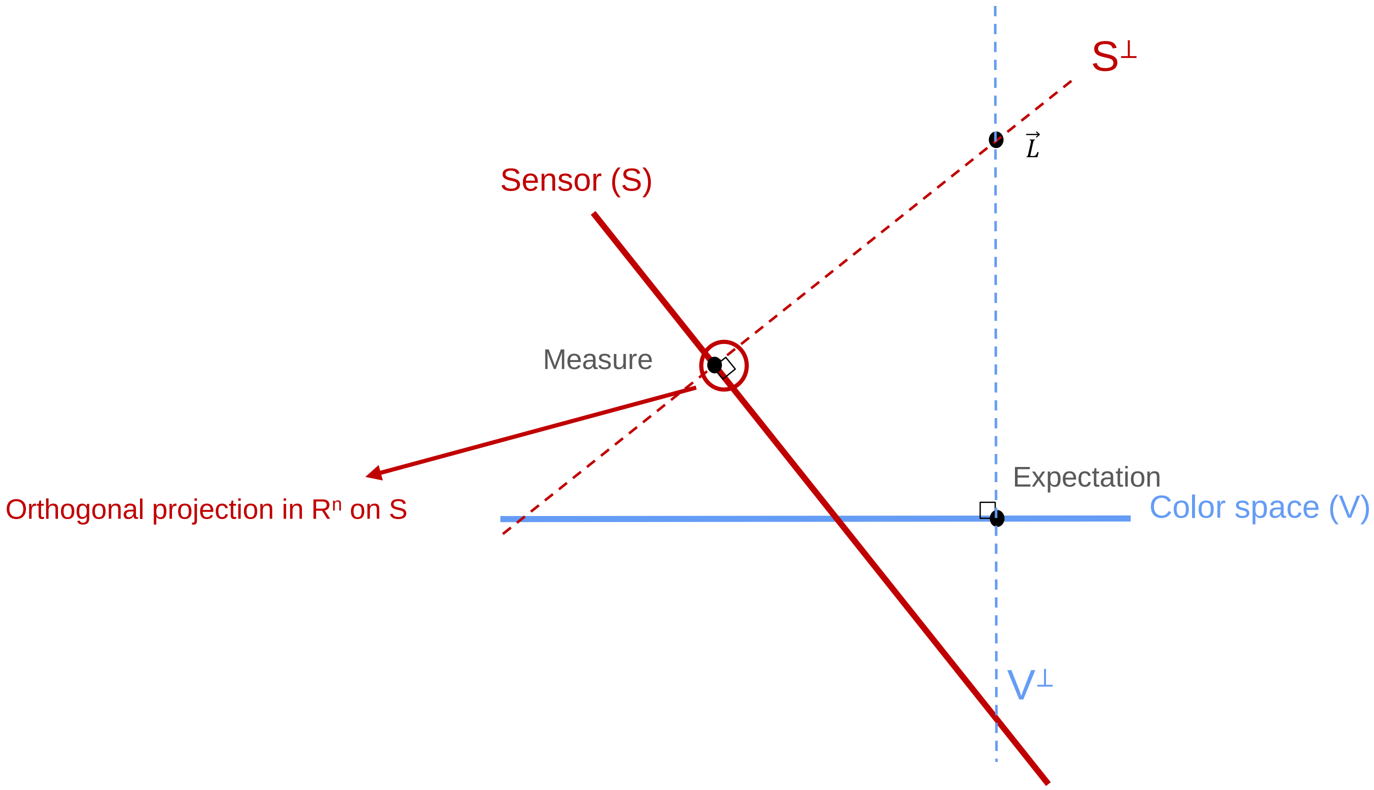
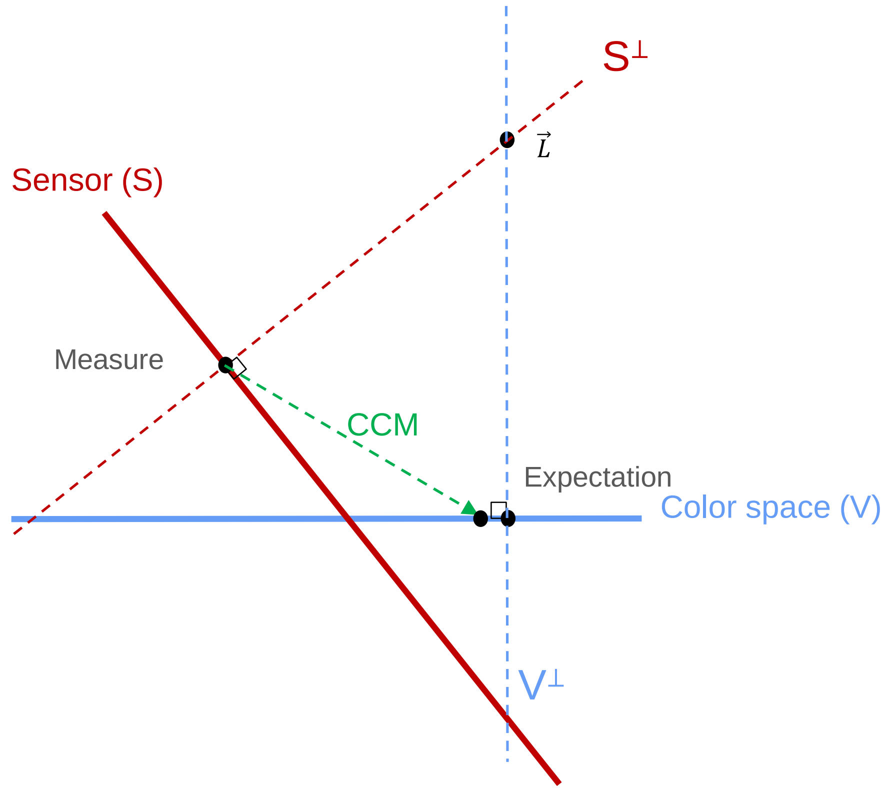


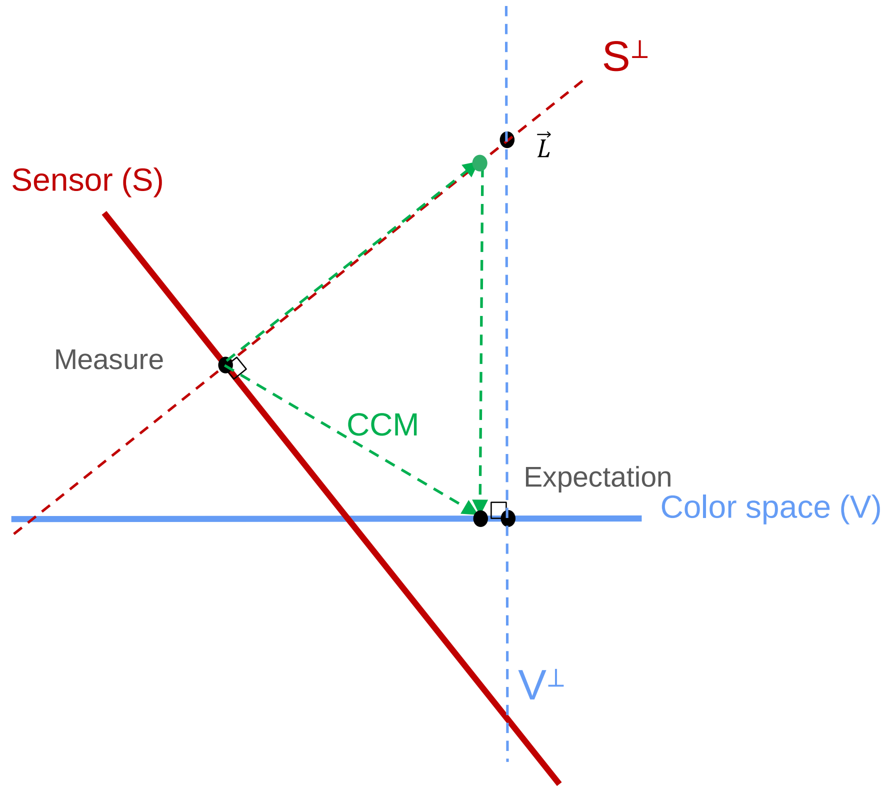
MATHEMATICAL REPRESENTATION OF LIGHT

- **Another schematic view of color correction:**
  - Application of the CCM:



MATHEMATICAL REPRESENTATION OF LIGHT

- **Another schematic view of color correction:**
  - Decomposition of the CCM:



MATHEMATICAL REPRESENTATION OF LIGHT

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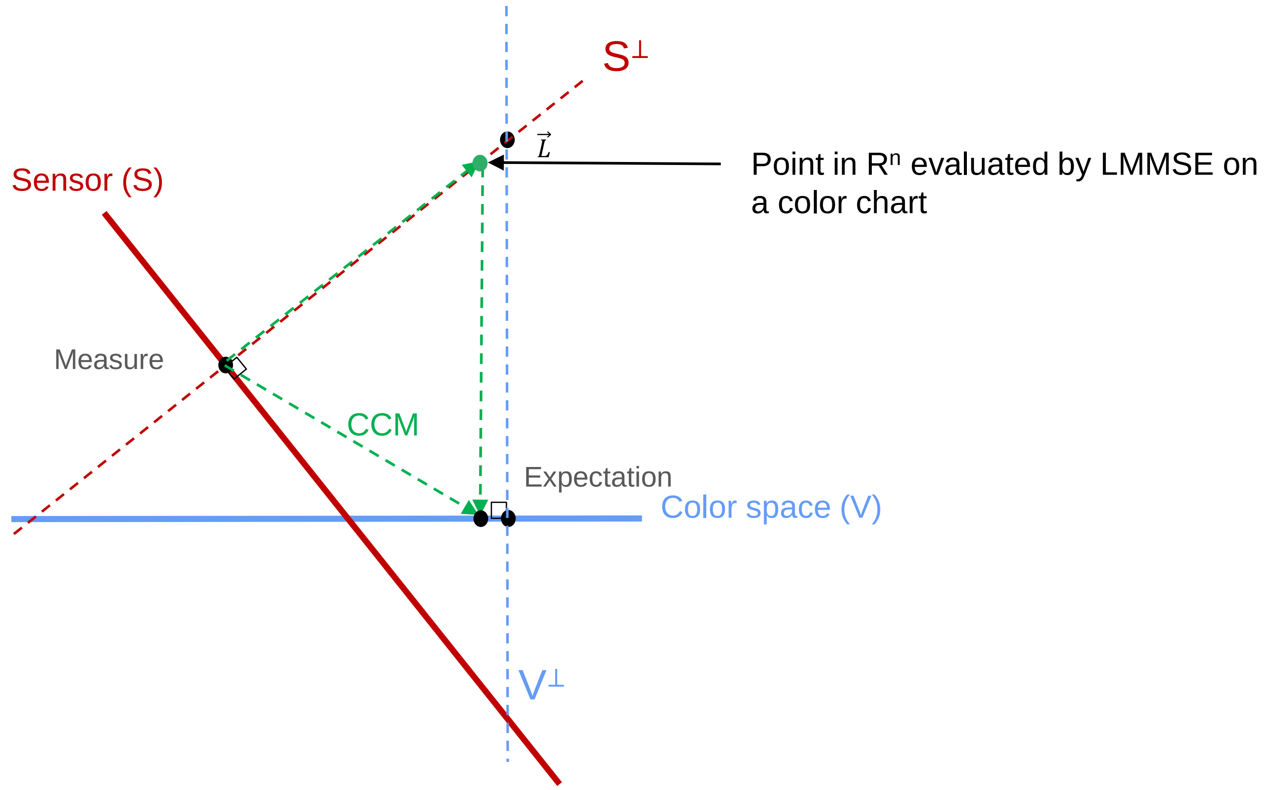
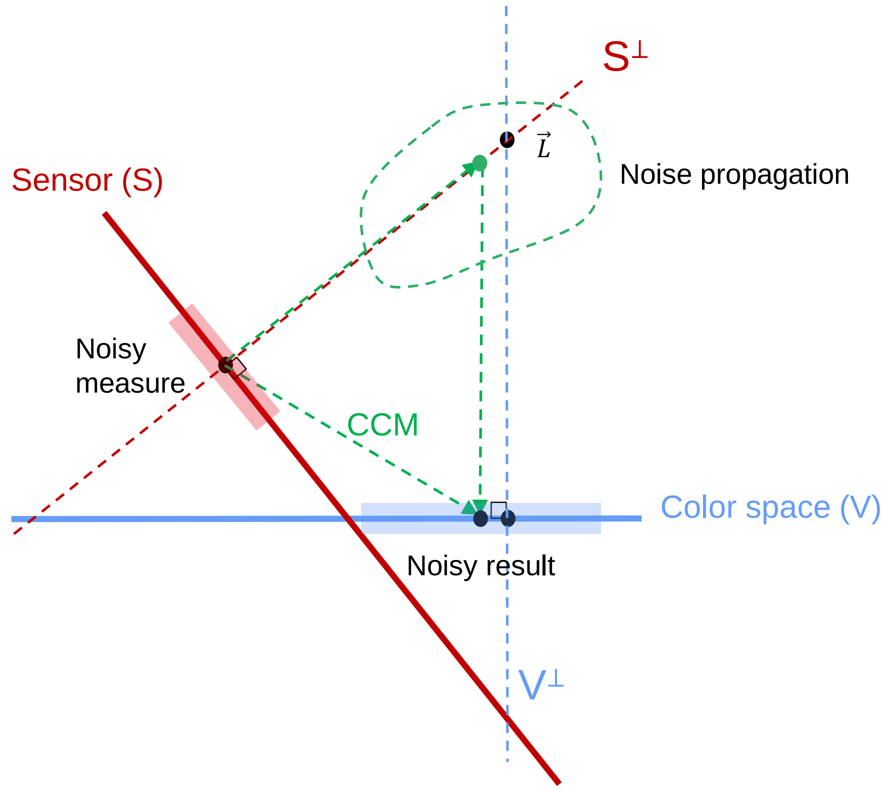


MATHEMATICAL REPRESENTATION OF LIGHT

- **Another schematic view of color correction:**
  - Noise and CCM:



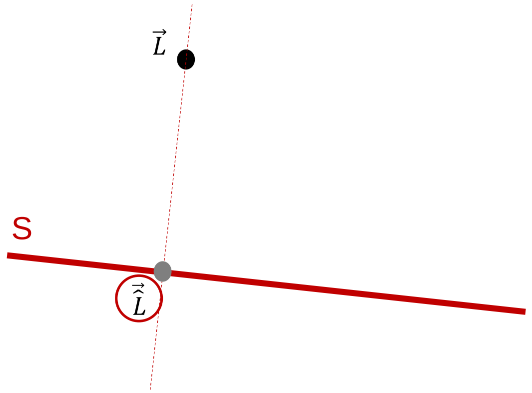
MATHEMATICAL REPRESENTATION OF LIGHT

- **Color correction Kernel (not scaled):**

  1) Radiance spectrum evaluation:

  - Cohen operator (explicit form):

$$\hat{L} = F.\left(F^T.F\right)^{-1}.F^T.L$$



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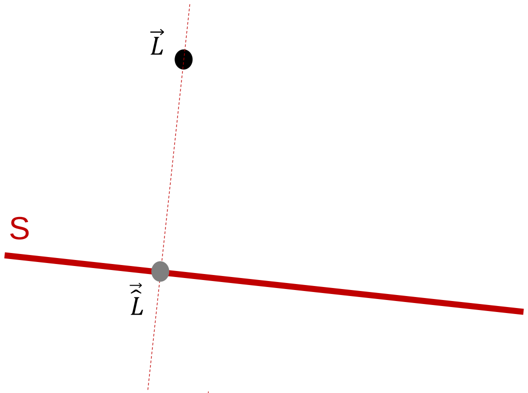
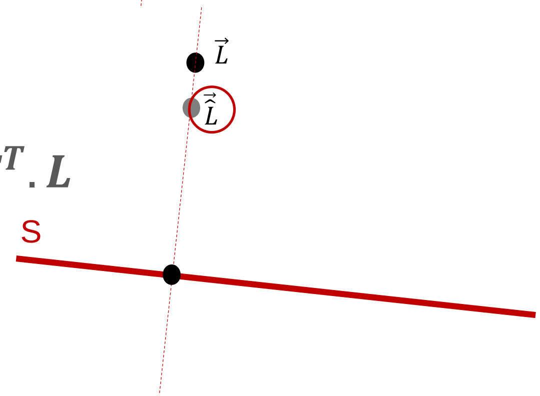


  - Use of a dataset (state of the art method):

$$\hat{L} = z_{set}.\left(F^T z_{set}\right)^T.\left(F^T z_{set}.\left(F^T z_{set}\right)^T\right)^{-1}.F^T.L$$

$n \times p$ matrix
with p data



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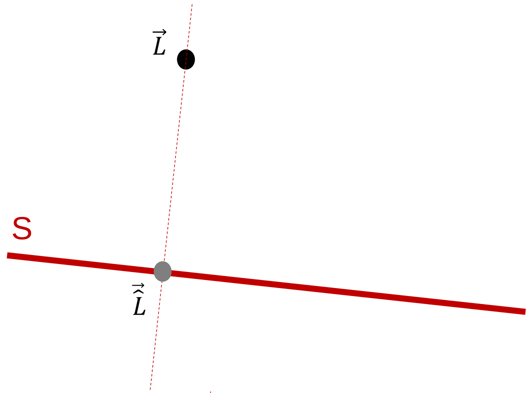

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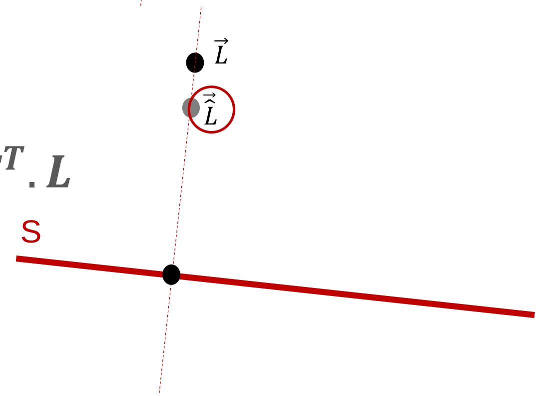
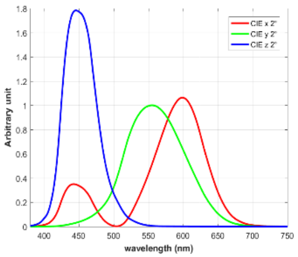


  2) Color space projection:

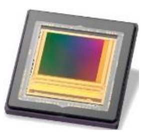
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{corrected} = H^T.\hat{L}$$



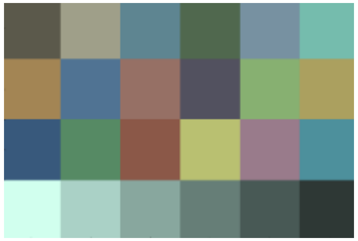
MATHEMATICAL REPRESENTATION OF LIGHT

- **State of the art (simplified):**
  - Classical color correction:

$$T = \begin{bmatrix} R_{T1} \dots R_{T24} \\ G_{T1} \dots G_{T24} \\ B_{T1} \dots B_{T24} \end{bmatrix}$$



$$S = \begin{bmatrix} R_{S1} \dots R_{S24} \\ G_{S1} \dots G_{S24} \\ B_{S1} \dots B_{S24} \end{bmatrix}$$



$$\widehat{M} = argmin_M(\|T - M.S\|^2)$$

$$\widehat{M} = T.S^T.(S.S^T)^{-1}$$

$$\widehat{M} = CCM.WB$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{corr} = \widehat{M} \begin{bmatrix} R \\ G \\ B \end{bmatrix}_{raw}$$

MATHEMATICAL REPRESENTATION OF LIGHT

- **Color correction matrix:**
  - Complete expression:

$$\hat{M} = C.H^T.z_{set}.(F^T z_{set})^T.(F^T z_{set}.(F^T z_{set})^T)^{-1}$$

Scaling matrix

Color space projection

Spectral evaluation operator

MATHEMATICAL REPRESENTATION OF LIGHT

- **Color correction matrix:**
  - Complete expression:

$$\widehat{M} = C.H^T.z_{set}.(F^T z_{set})^T.(F^T z_{set}.(F^T z_{set})^T)^{-1}$$

Scaling matrix   Color space projection   Spectral evaluation operator

  - Analogy with the state of the art:

$$\widehat{M} = C.H^T.z_{set}.(F^T z_{set})^T.(F^T z_{set}.(F^T z_{set})^T)^{-1}$$

« ∝ T »   « ∝ S »

MATHEMATICAL REPRESENTATION OF LIGHT

- **Color correction matrix:**
  - Complete expression:

$$\hat{M} = \underbrace{C}_{\text{Scaling matrix}}.\underbrace{H^T.z_{set}}_{\text{Color space projection}}.\underbrace{(F^T z_{set})^T.(F^T z_{set}.(F^T z_{set})^T)^{-1}}_{\text{Spectral evaluation operator}}$$

  - Analogy with the state of the art:

$$\hat{M} = C.\underbrace{H^T.z_{set}}_{\ll \propto T \gg}.(\underbrace{F^T z_{set}}_{\ll \propto S \gg})^T.(F^T z_{set}.(F^T z_{set})^T)^{-1}$$

  - Scaling: in 8-bits acquisition → display in sRGB standard

$$\underbrace{\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{sRGB}}_{[0\ ,\ 255]} = \hat{M}.\underbrace{\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{raw}}_{[0\ ,\ 255]}$$

PROSPECT AND CHALLENGE

- **Scaling:**
  - Important for compact writing of CCM: 3x3 fix matrix
  - Physical point of view: absolute scale → classical geometry
  - Perception point of view: relative scale → projective geometry?
    - To adapt formalism: homogenous coordinates
    - Homography as spectral color correction

CONCLUSION

- **Image sensor industry:**
  - Use of empirical color correction
  - Work in absolute physical units
- **Cognitive and mathematical color science:**
  - Mathematical approach
  - Relative scales
- **Strong link between domains useful to understand**

Thanks for attention!

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