ALGEBRA FOR COLOR CORRECTION IN IMAGE SENSORS
GENERAL CONTEXT

• PhD student at CEA-LETI (Grenoble) with LPNC:
  • Supervisors:
    • Jérôme Vaillant (CEA)
    • David Alleysson (LPNC)
  • Title: color restitution in low light level image sensors
  • Main deals:
    • Poor signal to noise ratio
    • Illumination conditions
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PLAN

• Introduction and context
  • The goal of color sensor design
  • Classical color correction (state of the art)

• Physics of light capture and sensor limitation
  • Photon absorption, noise

• Mathematical representation of light
  • Discrete approximation
  • Sensor space, color space
  • Color correction interpretation

• Challenge and prospects
INTRODUCTION AND CONTEXT

- **Color image sensor:**
  - Goal: acquire similar data as the human perception in given conditions:
**INTRODUCTION AND CONTEXT**

- **Color image sensor:**
  - Goal: acquire similar data as the human perception in given conditions:

- **Key components for color acquisition (recall):**

---

**Color sensor**

**Human visual system**

**Photodiode**

**Pixel**

**Color filters**

**IR cutoff filter**
INTRODUCTION AND CONTEXT

- **Color image sensor:**
  - Goal: acquire similar data as the human perception in given conditions:

- **Key components for color acquisition (recall):**
  - Photodiode
  - Pixel
  - Color filters
  - IR cutoff filter

Optimisation
INTRODUCTION AND CONTEXT

• Color image sensor:
  • Goal: acquire similar data as the human perception in given conditions:

  ![Color sensor](image1)
  ![Human visual system](image2)

• Key components for color acquisition (recall):

  ![Pixels](image3)
  ![Graph of spectral sensitivities](image4)
INTRODUCTION AND CONTEXT

• State of the art:
  • Basic image processing:

Demosaiçing
INTRODUCTION AND CONTEXT

• State of the art:
  • Basic image processing:

Demosaiçing  White balance

Illuminant
INTRODUCTION AND CONTEXT

- **State of the art:**
  - Basic image processing:

**Demosaiçcing**

**White balance**

**Color correction**

**Illuminant**

**QE**

**Color matching functions**
**INTRODUCTION AND CONTEXT**

- **State of the art:**
  - Classical color correction:

  \[
  T = \begin{bmatrix}
  R_{T1} & \cdots & R_{T24} \\
  G_{T1} & \cdots & G_{T24} \\
  B_{T1} & \cdots & B_{T24} \\
  1 & \cdots & 1
  \end{bmatrix}
  \]

  \[
  S = \begin{bmatrix}
  R_{S1} & \cdots & R_{S24} \\
  G_{S1} & \cdots & G_{S24} \\
  B_{S1} & \cdots & B_{S24} \\
  1 & \cdots & 1
  \end{bmatrix}
  \]

  \[
  \hat{M} = \arg\min_M (\|T - M.S\|^2)
  \]

  \[
  \hat{M} = T.S^T.(S.S^T)^{-1}
  \]

  \[
  \hat{M} = \begin{bmatrix}
  K & [V] \\
  0 & \cdots & 0 & 1
  \end{bmatrix}
  \]

  \[
  K = CCM.WB
  \]

  \[
  \begin{bmatrix}
  R \\
  G \\
  B_{offset}
  \end{bmatrix}
  = -K^T.(K.K^T)^{-1}.V
  \]

  \[
  \begin{bmatrix}
  R \\
  G \\
  B_{corr}
  \end{bmatrix}
  = \begin{bmatrix}
  C_{11} & C_{12} & C_{13} \\
  C_{21} & C_{22} & C_{23} \\
  C_{31} & C_{32} & C_{33}
  \end{bmatrix}
  \begin{bmatrix}
  W_{11} & 0 & 0 \\
  0 & W_{22} & 0 \\
  0 & 0 & W_{33}
  \end{bmatrix}
  \begin{bmatrix}
  R \\
  G \\
  B_{raw}
  \end{bmatrix}
  - \begin{bmatrix}
  R \\
  G \\
  B_{offset}
  \end{bmatrix}
  \]

  **CCM**

  **WB**
INTRODUCTION AND CONTEXT

• State of the art:
  • Exemple of color correction (computed on hyperspectral image)
  • CIE-D65 illuminant
  • Teledyne Onyx sensor with infrared cutoff filter (slides before)
State of the art:

- Exemple of color correction (computed on hyperspectral image):
- CIE-D65 illuminant
- Teledyne Onyx sensor with infrared cutoff filter (slides before)
PHYSICS OF LIGHT YIELDING AND SENSOR LIMITATION

- Photon absorption computation:
  - Physical units:
    - Black body 6500K
    - Reflectance rate $R_p(\lambda)$
    - Photon/electron conversion rate $QE(\lambda)$
    - Pixels
    - Physical absolute unit
• **Photon absorption computation:**
  • Electron measure:

  \[
  M_{e^-} = \frac{N_{lux} \cdot T_{int} \cdot a_{pix}^2}{4 \cdot f^2_\#} \int_0^\infty I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda
  \]

• **Limitation of a sensor, the noise:**
  • Noise: uncertainty on the electron number measurement
  • Main source of noise:
    • Readout noise (standard deviation given by the manufacturer, some electrons) Gaussian
    • Photonic shot noise \((\sigma_{ph}^2 = signal)\) Poissonian
Photon absorption computation:

Electron measure:

$$M_{e-} = \frac{N_{lux} \cdot T_{int} \cdot a_{pix}^2}{4 \cdot f^2_\#} \int_0^{\infty} I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda$$

Limitation of a sensor, the noise:

Noise: uncertainty on the electron number measurement

Main source of noise:

Readout noise (standard deviation given by the manufacturer, some electrons)
Gaussian

Photonic shot noise ($\sigma_{ph}^2 = signal$)
Poissonian

$$M_{e-} = \frac{N_{lux} \cdot T_{int} \cdot a_{pix}^2}{4 \cdot f^2_\#} \int_0^{\infty} I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda \pm \Delta M_{e-}$$
• **Color correction:**
  • Signal and noise amplification
  • Signal to noise ratio decrease
• **Color correction:**
  - Signal and noise amplification
  - Signal to noise ratio decrease

• **Quantitative exemple:**

  Usual model: « without correlation between channels »

![Graph showing SNR vs. illumination](image)
• Study of noise propagation $\rightarrow$ adapte spectral transmittances
  • Use large band filter for better raw SNR:

  \[ \text{Transmittance} \]

  \[ \lambda_1 \quad \lambda_n \]

  Better spectral property

  \[ \text{Transmittance} \]

  \[ \lambda_1 \quad \lambda_n \]

  Larger band $\rightarrow$ Higher raw SNR

• Final SNR increase? **Not obvious**
  • Noise can be amplified according to spectral shape and number of spectral channels
  • $\rightarrow$ We need an algebraic representation for noise propagation through color correction
ALGEBRAIC WRITTING

- From physics to algebraic representation:

\[ M_{e-} = \frac{N_{lux} \cdot T_{int} \cdot a_{pix}^2}{4 \cdot f_\#^2} \int_0^\infty I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda \]

Finite support

\[ M_{e-} = \frac{N_{lux} \cdot T_{int} \cdot a_{pix}^2}{4 \cdot f_\#^2} \int_{700nm}^{\infty} I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda \]

\[ M_{e-} \propto \int_{400nm}^{700nm} I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda \]
ALGEBRAIC WRITTING

- From physics to algebraic representation:

\[
M_e = \frac{N_{lux} \cdot T_{int} \cdot a_{pix}^2}{4 \cdot f_\#^2} \int_0^\infty I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda
\]

Finite support

\[
M_e = \frac{N_{lux} \cdot T_{int} \cdot a_{pix}^2}{4 \cdot f_\#^2} \int_{400 \text{nm}}^{700 \text{nm}} I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda
\]

\[
M_e \propto \int_{400 \text{nm}}^{700 \text{nm}} I_{lum}(\lambda) \cdot R_p(\lambda) \cdot QE(\lambda) \cdot d\lambda
\]

Discretisation:

\[
M_e \propto \sum_{k=1}^{n} I_{lum}(\lambda_k) \cdot R_p(\lambda_k) \cdot QE(\lambda_k) \cdot \Delta \lambda
\]

Matrix form

\[
M_e \propto QE^T \cdot diag(I_{lum}) \cdot R_p
\]
MATHEMATICAL REPRESENTATION OF LIGHT

- **Discrete approximation of physics:**
  - Measure or digital interpolation:

  \[
  \begin{align*}
  &\text{Physical function } f(\lambda) \\
  &\text{Measure distribution } \varphi_1(\lambda) \\
  &\lambda_1 \quad \lambda_n \\
  &\lambda(\text{nm})
  \end{align*}
  \]

  Vectorized function \( f \)

  \[
  \begin{align*}
  f(\lambda_1) &= (f \ast \varphi_1)(\lambda) \\
  f(\lambda_2) &= (f \ast \varphi_2)(\lambda) \\
  \vdots \\
  f(\lambda_n) &= (f \ast \varphi_n)(\lambda)
  \end{align*}
  \]
• **Discrete approximation of physics:**
  • Measure or digital interpolation:

  \[ f \in R^n \]
  \[ f = \sum_{k=1}^{n} f_k \cdot e_k \]

  \[ [e_1, \ldots, e_n] = [(1,0, \ldots, 0), \ldots, (0, \ldots, 0,1)] \]

• **Vector representation:**
  • \( n \)-space \( \rightarrow R^n \) generated by the canonic base: a finite dimension Hilbert space

\[
\begin{array}{c}
\text{Vectorized function } f \\
\hline
f(\lambda_1) = (f \ast \varphi_1)(\lambda) \\
f(\lambda_2) \\
\vdots \\
f(\lambda_n) = (f \ast \varphi_n)(\lambda)
\end{array}
\]
MATHEMATICAL REPRESENTATION OF LIGHT

- Discrete approximation of physics:
  - Formalism recalling:
    - Consider $f(\lambda)$, a spectral function
    - $\tilde{f}$ is the vectorized form of $f$

Integration in pre-Hilbert space:

$$\int_{\lambda_1}^{\lambda_n} f(\lambda) \cdot d\lambda \approx \sum_{k=1}^{n} f(\lambda_k) \cdot \Delta \lambda = [\tilde{f} \cdot \mathbf{1}] \cdot \Delta \lambda$$
MATHEMATICAL REPRESENTATION OF LIGHT

- **Discrete approximation of physics:**
  - Formalism recalling:

  Vectorized function $f$  
  \[
  \begin{array}{c|c}
  f(\lambda_1) & g(\lambda_1) \\
  f(\lambda_2) & g(\lambda_2) \\
  \vdots & \vdots \\
  f(\lambda_n) & g(\lambda_n)
  \end{array}
  \]

  Matrix writing:
  \[
  a = \hat{f} \cdot \hat{g} = f^T g
  \]

- For vectorial study no need to scale the scalar products (physical parameters of $\Delta \lambda$)
• **Sensor algebraic space:**
  
  • Consider an RGB image sensor having spectral channels such as $(\vec{r}, \vec{g}, \vec{b})$ are $R^n$ elements.
    
    • With: $\vec{r} = \sum_{k=1}^n r_k \cdot \vec{e}_k$, $\vec{g} = \sum_{k=1}^n g_k \cdot \vec{e}_k$, $\vec{b} = \sum_{k=1}^n b_k \cdot \vec{e}_k$

  • This family is free and generates a 3 dimensions $R^n$ subspace:

  ![Diagram of Sensor^3 in R^n](image-url)
• **Color (or display) algebraic space:**
  • Consider a standard color space such as XYZ generated in $R^n$ by $(\tilde{x}, \tilde{y}, \tilde{z})$.
  • With: $\tilde{x} = \sum_{k=1}^{n} x_k \cdot \vec{e}_k$, $\tilde{y} = \sum_{k=1}^{n} y_k \cdot \vec{e}_k$, $\tilde{z} = \sum_{k=1}^{n} z_k \cdot \vec{e}_k$
  • This family is free and generates a 3 dimensions $R^n$ subspace:
• Schematic radiance projection:
  • \( L \) radiance spectrum, \( F \) quantum efficiencies, \( H \) color matching functions

Measure operator:
\[
F = \begin{bmatrix}
R_1 & G_1 & B_1 \\
\vdots & \vdots & \vdots \\
R_n & G_n & B_n
\end{bmatrix}
\]

Measure = \( F^T \cdot L \)
MATHEMATICAL REPRESENTATION OF LIGHT

• Schematic radiance projection:
  • $L$ radiance spectrum, $F$ quantum efficiencies, $H$ color matching functions

Measure operator:

$$ F = \begin{bmatrix} R_1 & G_1 & B_1 \\ R_n & G_n & B_n \end{bmatrix} $$

$$ Measure = F^T \cdot L $$

Expectation operator:

$$ H = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_n & Y_n & Z_n \end{bmatrix} $$

$$ Expectation = H^T \cdot L $$
MATHEMATICAL REPRESENTATION OF LIGHT

- **Schematic radiance projection:**
  - $L$ radiance spectrum, $F$ quantum efficiencies, $H$ color matching functions

Measure operator:

$$ F = \begin{bmatrix} R_1 & G_1 & B_1 \\ R_n & G_n & B_n \end{bmatrix} $$

$Measure = F^T . L$

Expectation operator:

$$ H = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_n & Y_n & Z_n \end{bmatrix} $$

$Expectation = H^T . L$
• Another schematic view of color correction:
  • Raw acquisition:
Another schematic view of color correction:

- Raw acquisition:
• Another schematic view of color correction:
  • Application of the CCM:
• Another schematic view of color correction:
  • Decomposition of the CCM:
Another schematic view of color correction:

- Decomposition of the CCM:
MATHEMATICAL REPRESENTATION OF LIGHT

- Another schematic view of color correction:
  - Noise and CCM:
 MATHEMATICAL REPRESENTATION OF LIGHT

- **Color correction Kernel (not scaled):**
  1) Radiance spectrum evaluation:
    - Cohen operator (explicit form):
      \[
      \hat{L} = F \cdot (F^T F)^{-1} \cdot F^T \cdot L
      \]
• **Color correction Kernel (not scaled):**
  1) Radiance spectrum evaluation:

  • Cohen operator (explicit form):

    \[ \hat{L} = F \cdot (F^T \cdot F)^{-1} \cdot F^T \cdot L \]

  • Use of a dataset (state of the art method):

    \[ \hat{L} = z_{set} \cdot (F^T z_{set})^T \cdot (F^T z_{set} \cdot (F^T z_{set})^T)^{-1} \cdot F^T \cdot L \]
**Color correction Kernel (not scaled):**

1) Radiance spectrum evaluation:

   - Cohen operator (explicit form):
     \[ \hat{L} = F \cdot (F^T F)^{-1} \cdot F^T \cdot L \]
   - Use of a dataset (state of the art method):
     \[ \hat{L} = z_{set} \cdot (F^T z_{set})^T \cdot (F^T z_{set} \cdot (F^T z_{set})^T)^{-1} \cdot F^T \cdot L \]

2) Color space projection:

\[ \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{corrected}} = H^T \cdot \hat{L} \]
• State of the art (simplified):
  • Classical color correction:

\[
T = \begin{bmatrix}
R_{T1} & \ldots & R_{T24} \\
G_{T1} & \ldots & G_{T24} \\
B_{T1} & \ldots & B_{T24}
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
R_{S1} & \ldots & R_{S24} \\
G_{S1} & \ldots & G_{S24} \\
B_{S1} & \ldots & B_{S24}
\end{bmatrix}
\]

\[
\hat{M} = \arg\min_{M} (\|T - M.S\|^2)
\]

\[
\hat{M} = T.S^T.(S.S^T)^{-1}
\]

\[
\hat{M} = CCM.WB
\]

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{corr} = \hat{M} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{raw}
\]
MATHEMATICAL REPRESENTATION OF LIGHT

• Color correction matrix:
  • Complete expression:

\[ \hat{M} = C \cdot H^T \cdot z_{set} \cdot (F^T z_{set})^T \cdot (F^T z_{set} \cdot (F^T z_{set})^T)^{-1} \]
MATHEMATICAL REPRESENTATION OF LIGHT

• Color correction matrix:
  • Complete expression:

\[ \hat{M} = C \cdot H^T \cdot z_{set} \cdot (F^T z_{set})^T \cdot (F^T z_{set} \cdot (F^T z_{set})^T)^{-1} \]

Scaling matrix  Color space projection  Spectral evaluation operator

• Analogy with the state of the art:

\[ \hat{M} = C \cdot H^T \cdot z_{set} \cdot (F^T z_{set})^T \cdot (F^T z_{set} \cdot (F^T z_{set})^T)^{-1} \]

« \( \propto T \) »  « \( \propto S \) »
**Color correction matrix:**

- **Complete expression:**

\[
\hat{M} = C \cdot H^T \cdot z_{\text{set}} \cdot (F^T z_{\text{set}})^T \cdot (F^T z_{\text{set}} \cdot (F^T z_{\text{set}})^T)^{-1}
\]

- **Analogy with the state of the art:**

\[
\hat{M} = C \cdot H^T \cdot z_{\text{set}} \cdot (F^T z_{\text{set}})^T \cdot (F^T z_{\text{set}} \cdot (F^T z_{\text{set}})^T)^{-1}
\]

- **Scaling:** in 8-bits acquisition → display in sRGB standard

\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{\text{raw}} = \hat{M} \cdot \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{\text{sRGB}}
\]

\([0, 255]\)
PROSPECT AND CHALLENGE

• Scaling:
  • Important for compact writing of CCM: 3x3 fix matrix
  • Physical point of view: absolute scale → classical geometry
  • Perception point of view: relative scale → projective geometry?
    • To adapt formalism: homogenous coordinates
    • Homography as spectral color correction
CONCLUSION

• Image sensor industry:
  • Use of empirical color correction
  • Work in absolute physical units

• Cognitive and mathematical color science:
  • Mathematical approach
  • Relative scales

• Strong link between domains useful to understand
Thanks for attention!