'An original approach to color properties of surfaces, as sensed by the human eye'

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Introduction

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Color naming, unique hues, and hue cancellation predicted from singularities in reflection properties

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AN ILLUMINANT-INDEPENDENT ANALYSIS OF REFLECTANCE AS SENSED BY HUMANS, AND ITS APPLICABILITY TO COMPUTER VISION

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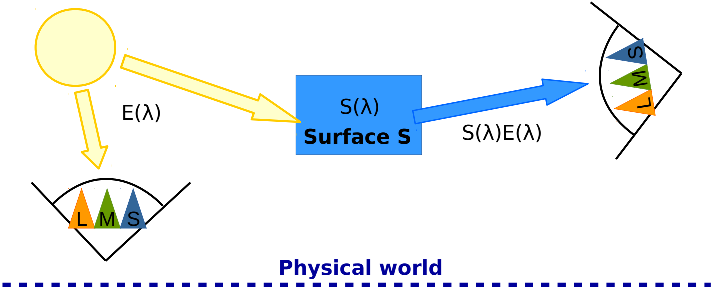
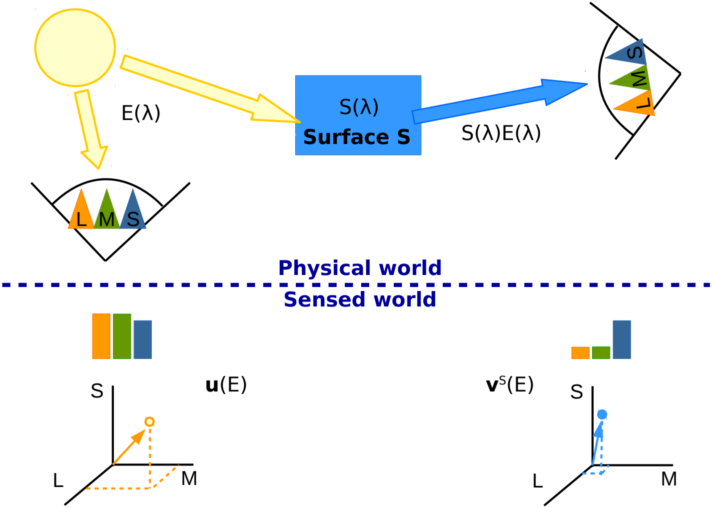
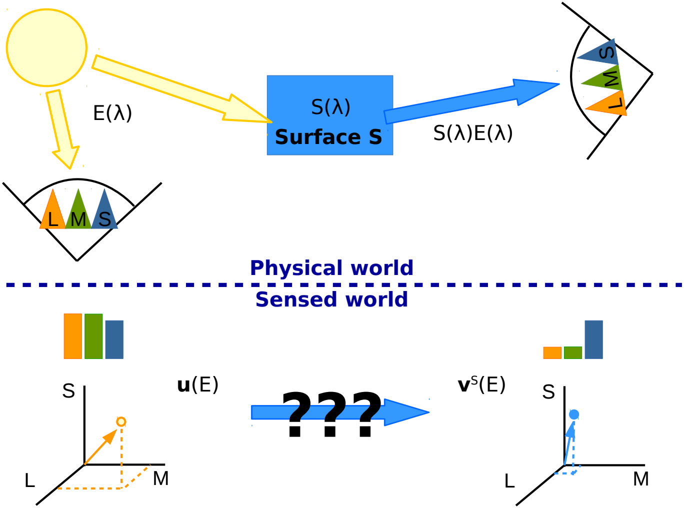
Sensed reflectance?



Sensed reflectance?



Sensed reflectance?



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Sensed space

Space of three dimensions

- Three types of human cones, with different *sensitivity functions* $\mathbf{R}(\lambda) = (R_L(\lambda), R_M(\lambda), R_S(\lambda))$.
- Sensed incident light:

$$\mathbf{u}(E) = \begin{pmatrix} u_L(E) \\ u_M(E) \\ u_S(E) \end{pmatrix} = \int_\Lambda E(\lambda)\mathbf{R}(\lambda)d\lambda.$$

- Sensed reflected light:

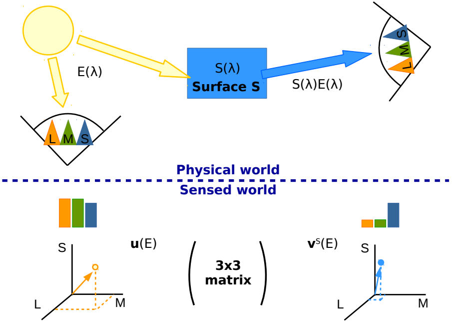
$$\mathbf{v}^{\mathbb{S}}(E) = \begin{pmatrix} v_L^{\mathbb{S}}(E) \\ v_M^{\mathbb{S}}(E) \\ v_S^{\mathbb{S}}(E) \end{pmatrix} = \int_\Lambda E(\lambda)\mathbf{R}(\lambda)S(\lambda)d\lambda.$$

- Projection from a space of infinite dimension into a three dimensional space

Sensed reflectance

- Philipona & O'Regan: the sensed analogue of reflectance is a **linear operator**, **illuminant-independent**:

$$\mathbf{v}^{\mathbb{S}}(E) = A^{\mathbb{S}}\mathbf{u}(E)$$

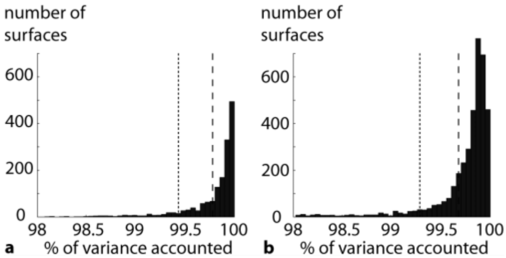


- The 3x3 *Reflectance Matrix* $A^{\mathbb{S}}$ represents the linear operator.

Computation of the Reflectance Matrices

- 1600 munsell chips and 3000 natural surfaces.
  Matrices $A^{\mathbb{S}}$ computed for each surfaces, through a linear regression over a set of **natural illuminants** (500).

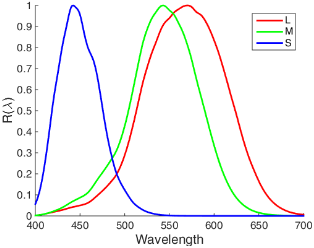
$$\mathbf{v}^{\mathbb{S}}(E) = A^{\mathbb{S}}\mathbf{u}(E)$$



- Sensed analogue of surface reflectance is very well modeled by a linear operator!
  $\Rightarrow$ properties of the $A^{\mathbb{S}}$ account for reflectance properties of surfaces surfaces, as sensed the by human eye.

Non-diagonal Reflectance Matrices

- We would like to study the properties of $A^{\mathbb{S}}$.
- However:



L and M cones are overlapping $\Rightarrow$ $A^{\mathbb{S}}$ is most often not diagonal:

$$\begin{pmatrix} v_L^{\mathbb{S}}(E) \\ v_M^{\mathbb{S}}(E) \\ v_S^{\mathbb{S}}(E) \end{pmatrix} = \begin{pmatrix} 0.2590 & 0.2151 & -0.0136 \\ -0.0979 & 0.5861 & -0.0090 \\ -0.0099 & 0.0155 & 0.4212 \end{pmatrix} \begin{pmatrix} u_L(E) \\ u_M(E) \\ u_S(E) \end{pmatrix}.$$

- There are 9 reflection coefficients $\Rightarrow$ need to **lower the number** of coefficients !

Diagonalization of the Reflectance Matrix

- A solution is the matrix **diagonalization**.
- Philipona & O'Regan showed all Reflectance Matrices can be diagonalized by a per surface transformation: : $A^{\mathbb{S}} = \left(T^{\mathbb{S}}\right)^{-1} D^{\mathbb{S}} T^{\mathbb{S}}$.

$$
\begin{aligned}
& \mathbf{v}^{\mathbb{S}}(E) = A^{\mathbb{S}}\mathbf{u}(E) \\
\Leftrightarrow\ & \mathbf{v}^{\mathbb{S}}(E) = \left(T^{\mathbb{S}}\right)^{-1} D^{\mathbb{S}} T^{\mathbb{S}}\mathbf{u}(E), \\
\Leftrightarrow\ & T^{\mathbb{S}}\mathbf{v}^{\mathbb{S}}(E) = D^{\mathbb{S}} T^{\mathbb{S}}\mathbf{u}(E), \\
\Leftrightarrow\ & \begin{pmatrix} \tilde{v}^{\mathbb{S}}{}_{\tilde{L}}(E) \\ \tilde{v}^{\mathbb{S}}{}_{\tilde{M}}(E) \\ \tilde{v}^{\mathbb{S}}{}_{\tilde{S}}(E) \end{pmatrix} = \begin{pmatrix} r^{\mathbb{S}}_{\tilde{L}} & 0 & 0 \\ 0 & r^{\mathbb{S}}_{\tilde{M}} & 0 \\ 0 & 0 & r^{\mathbb{S}}_{\tilde{S}} \end{pmatrix} \begin{pmatrix} \tilde{u}^{\mathbb{S}}{}_{\tilde{L}}(E) \\ \tilde{u}^{\mathbb{S}}{}_{\tilde{M}}(E) \\ \tilde{u}^{\mathbb{S}}{}_{\tilde{S}}(E) \end{pmatrix},
\end{aligned}
$$

- We now have only **three independent reflection coefficients** $r_i^{\mathbb{S}}$!

Singularities

- Some Reflectance Matrices are particular: they are singular.
- Two kind of singularities:
  - First kind is when the three reflection coefficients $r_i^{\mathbb{S}}$ are about equal.
  - Second kind is when one reflection coefficient is either very large or very small compared to the other two.
- Similarly to achromatic surfaces, singular surfaces of second kind are expected to have a particular perceptual status.

Singularity index

- Philipona & O'Regan defined a measure of this second kind of singularity with a singularity index:

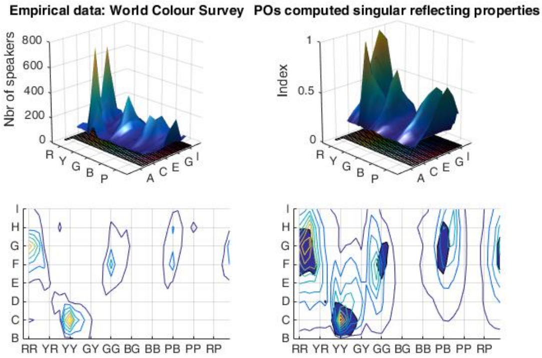
$$\sigma^{\mathbb{S}} = max\left(\frac{\sigma_1^{\mathbb{S}}}{\sigma_1^{max}}, \frac{\sigma_2^{\mathbb{S}}}{\sigma_2^{max}}\right),$$

- where, if the reflection coefficients are ordered decreasingly ${r_1}^{\mathbb{S}} > {r_2}^{\mathbb{S}} > {r_3}^{\mathbb{S}}$:
    - $\sigma_1^{\mathbb{S}} = {r_1}^{\mathbb{S}} / {r_2}^{\mathbb{S}}$
    - $\sigma_2^{\mathbb{S}} = {r_2}^{\mathbb{S}} / {r_3}^{\mathbb{S}}$
    - σ_1^{max} and σ_2^{max} are the maximum $\sigma_1^{\mathbb{S}}$ and $\sigma_2^{\mathbb{S}}$ values respectively over the entire dataset of surfaces.

- The singularity index will thus be high in cases:
    - ${r_1}^{\mathbb{S}} >> {r_2}^{\mathbb{S}} \simeq {r_1}^{\mathbb{S}}$ (high σ_1).
    - ${r_1}^{\mathbb{S}} \simeq {r_2}^{\mathbb{S}} >> {r_1}^{\mathbb{S}}$ (high σ_2).

Singularities and focal colors

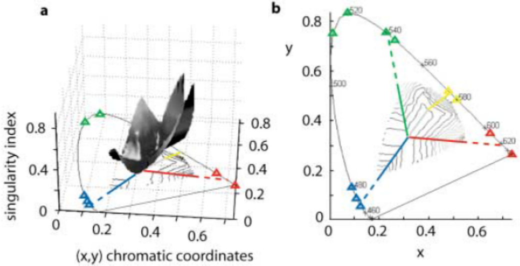
- Philipona & O'Regan showed that singularities correlate with focal colors: Berlin and Kay, (1969).



r = 0.64.

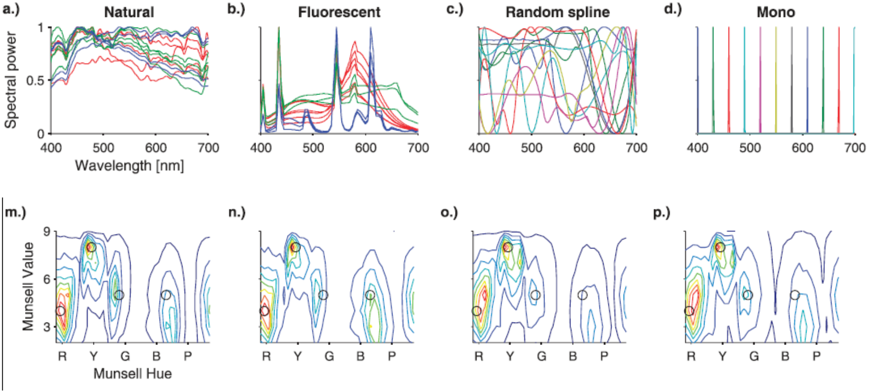
Singularities and unique hues

- Philipona & O'Regan showed that singularities may be related to unique hues: Kuehni, (2004).



Singularities and illuminants

- Model is accurate for natural, braod illuminants. What about non-natural illuminants? Witzel et al., (2015)



In short

The linear model of Philipona & O'Regan is very simple, yet very precise. It also may give a hint to explain the particular perceptual status of focal colors and unique hues.

But...

Illuminant-Independent hypothesis and implementation over a database

- Operators are not really illuminant-independent: $\Rightarrow$ Obtained through linear regression over a **set of natural illuminants**.
- A per surface diagonalization, unlikely to happen in our neural system.

$\Rightarrow$

- We would like to compute the operators in a very **illuminant-independent**. fashion
- We would like to have one global transformation diagonalizing all operators.

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Novel Illuminant-Independent approach

- New method to compute the $A^{\mathbb{S}}$ fully independently with respect to the illuminant:

$$\begin{aligned} & \mathbf{v}^{\mathbb{S}}(E) = A^{\mathbb{S}}\mathbf{u}(E) \\ \Leftrightarrow\ & \mathbf{v}^{\mathbb{S}}(E) = \int_{\Lambda} E(\lambda) A^{\mathbb{S}} \mathbf{R}(\lambda) d\lambda, \\ \Leftrightarrow\ & \int_{\Lambda} E(\lambda) \left(S(\lambda) \cdot \mathbf{R}(\lambda) - A^{\mathbb{S}} \mathbf{R}(\lambda) \right) d\lambda = 0. \end{aligned}$$

- Which, according to the *Fundamental Lemma of Calculus of variation* is equivalent to:

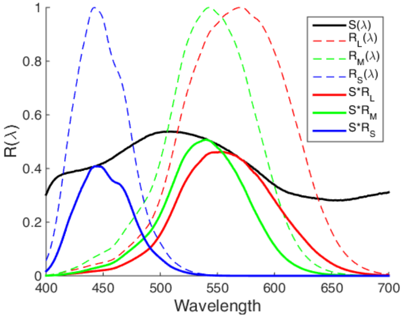
$$S(\lambda) \cdot \mathbf{R}(\lambda) = A^{\mathbb{S}} \mathbf{R}(\lambda) \qquad \forall \lambda \in \Lambda.$$

- This time, the multi-linear regression can be performed on λ!

Novel Illuminant-Independent approach

$$S(\lambda) \cdot \mathbf{R}(\lambda) = A^{\mathbb{S}}\mathbf{R}(\lambda) \qquad \forall \lambda \in \Lambda.$$

- In this case, reflectance acts as a transformation of the cone sensitivities:



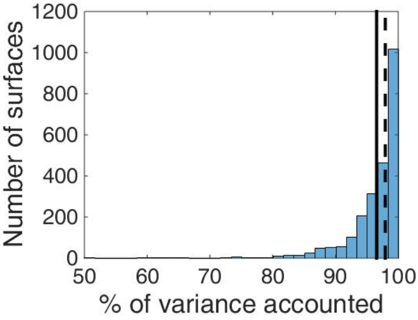
- Linear operator: fit the transformed cone sensitivities with a linear combination of the initial ones.

Validity of the Illuminant-Independent hypothesis

- Is the illuminant-independence hypothesis valid?

$$S(\lambda) \cdot \mathbf{R}(\lambda) = A^{\mathbb{S}}\mathbf{R}(\lambda) \qquad \forall \lambda \in \Lambda.$$

$\Rightarrow$ Compute the percentage of variance accounted for by the model (Data: 1600 munsell chips, 800 natural reflectances):



- mean: 97% and median: 98%
- Illuminant-Independent hypothesis is valid for most of the surfaces, but not all of them.

Quantitative comparison with Philipona & O'Regan's approach

- Linear relation between incident and reflected sensed light still satisfied?

$$\mathbf{v}^{\mathbb{S}}(E) = A^{\mathbb{S}}\mathbf{u}(E)$$

- Test on natural illuminants: (400 illuminants).

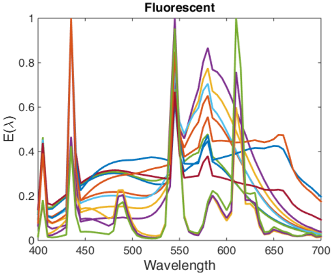
Illuminants	Variance accounted by the model for whole set of surfaces					
	With the II approach			With PO's approach		
	mean	median	min	mean	median	min
natural	**99.70**	**99.96**	**91.93**	**99.94**	**99.97**	**98.83**

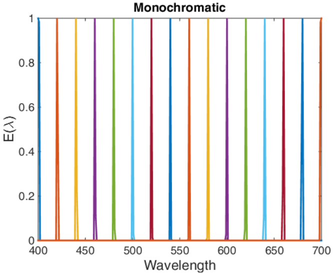
- The new Reflectance Matrices are consistent with the linear relation. Quite surprising!
- Philipona & O'Regan's matrices allow a better approximation than the II approach if computed and tested with the same natural illuminant dataset.

- What about un-natural illuminants?

Quantitative comparison with Philipona & O'Regan's approach

- Un-natural illuminants obtained from Witzel et al., (2015).



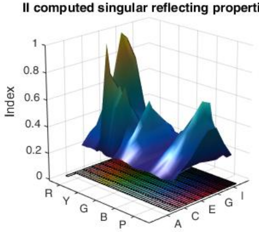


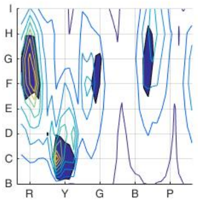
Quantitative comparison with Philipona & O'Regan's approach

Illuminants	Variance accounted by the model for all set of surfaces					
	With the II approach			With PO's approach		
	mean	median	min	mean	median	min
natural	99.70	99.96	91.93	99.94	99.97	98.83
fluorescent	**99.45**	**99.75**	**87.23**	**98.69**	**99.45**	**65.53**
monochromatic	**96.33**	**97.83**	**37.72**	**91.68**	**96.00**	**-2.30**

- The new Reflectance Matrices are more accurate!
- Thus:
  - more robust than Philipona & O'Regan's ones to illuminant change.
  - directly refer to the action of a surface such as it is defined by the model.

Singularities of new matrices





r= 0.61 $\Rightarrow$ the correlation is preserved.

In short

- The illuminant-independent approach is less costly than Philipona & O'Regan's one.
- It is a direct consequence of the Illuminant-Independent hypothesis of the sensed reflectance.
- New insight into the limits of the model .
- The resulting Reflectance Matrices are more in agreement with the definition of sensed reflectance.
- The model is now more robust to illuminant change.
- The new matrices are a good groundwork to finding a global transformation.

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Color constancy and Philipona & O'Regan's model

- Color constancy: discount the effect of the illuminant to access intrinsic reflection properties of surfaces.

$$\mathbf{v}^{\mathbb{S}}(E) = A^{\mathbb{S}}\mathbf{u}(E)$$

- To have an easier access to reflection properties: matrix diagonalization.

$$A^{\mathbb{S}} = \left(T\right)^{-1} D^{\mathbb{S}} T$$

- The compatibility requires a joint-diagonalization of every matrices by global transformation T.
- Vazquez-Corral et al., (2012) computed a global transformation T similar to a joint diagonalization. But:
  - Complicated algorithm: spherical sampling (Finlayson et Süsstrunk, 2001)
  - requires a database of illuminants.
  - requires the specification of an illuminant of reference.

Now that we have Illuminant-Independent Reflectances Matrices, we can use a gradient descent without the specification of any illuminant!

Gradient descent

- Commonly used measure of the diagonality of a matrix:

$$JD = \frac{1}{N}\sum_{k=1}^{N}\sum_{i\neq j}\left|M_{ij}^{k}\right|^{2}$$

- We want to find the transformation T_{opt} that minimizes this measure:

$$T_{\text{opt}} = \arg\min_{T\in\mathbb{R}^{3\times 3}} JD(T)$$

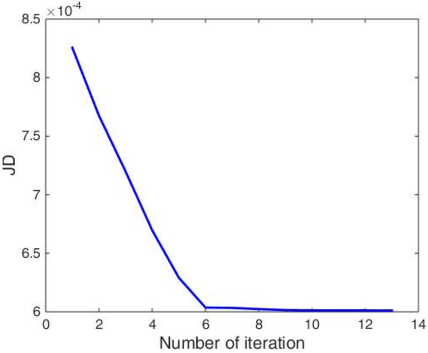
- The gradient formula was taken from Hori, (1999):

$$\nabla JD(T) = 2T\sum_{k=1}^{N}\left[\left(T^{-1}A^{\mathbb{S}_k}T\right)^{t}, \left(T^{-1}A^{\mathbb{S}_k}T - \text{diag}(T^{-1}A^{\mathbb{S}_k}T)\right)\right],$$

- The discrete gradient descent is thus:

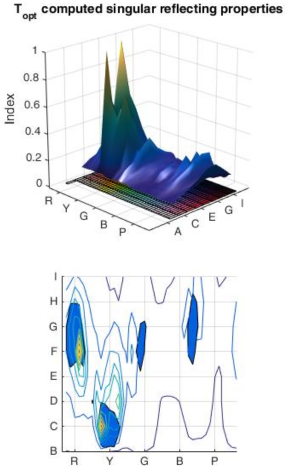
$$T_{n+1} = T_n - \alpha_n \frac{\nabla JD(T_n)}{\|\nabla JD(T_n)\|}, \quad n \geq 0.$$

Gradient descent (results)



- The off diagonal elements of a transformed Reflectance Matrix are in average 50x smaller than the diagonal elements.
  $\Rightarrow$ Empirical evidence that T almost diagonalizes all of our matrices.

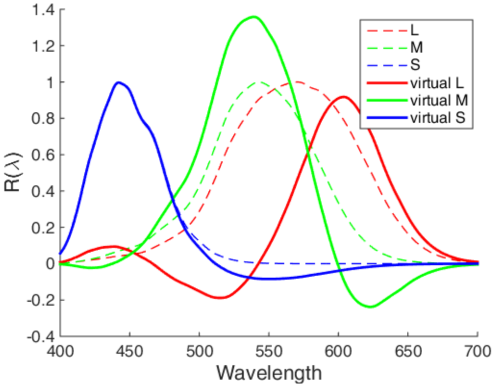
Singularities with global transformation



r= 0.56 ⇒ the correlation is preserved.

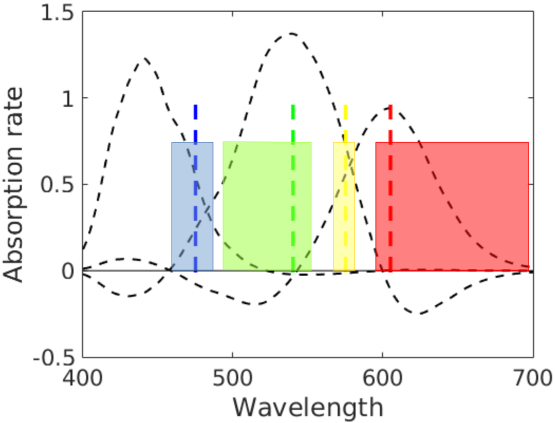
Virtual sensors

- A global transformation on the cone inputs is equivalent to a global transformation applied on the cone sensitivity functions, resulting in *optimal virtual sensors*.



Virtual sensors and unique hues

- interestingly, the virtual sensor's peaks and crossings correlate greatly with empirical findings on unique hues: Kuehni, (2004):



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Conclusion: in short

- An original approach to color vision.
- Does not require any database of illuminants.
  $\Rightarrow$ More consistent with the illuminant-Independent hypothesis of the model and more robust to illuminant change.
- Allow simpler computation of a global transformation T compatible with classic approaches on color constancy.
- Singularities found with the Reflectance Matrices, as well as optimal 'virtual' sensors, may give a piece of explanation to human perceptual phenomena such as the existance of color categories and unique hues.

Thanks

Thank you for your attention

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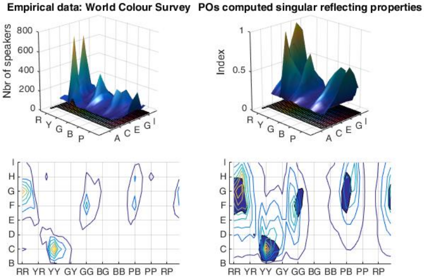
Singularities

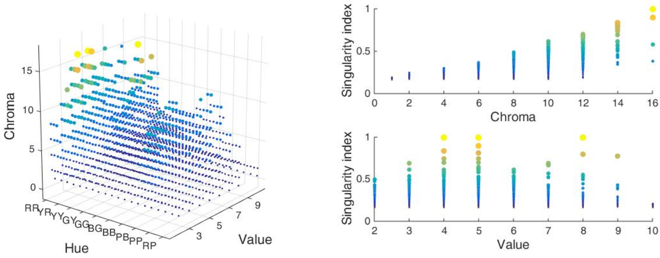
- Singularities in sensitive reflectances are correlated with the particularity of some surface colors to be perceived as focal:



- The Munsell color system divides colors according to 3 characteristics: hue, value (lightness) and chroma (saturation)
- How these last two correlate with singularities ?

Properties of singularity index: value and chroma

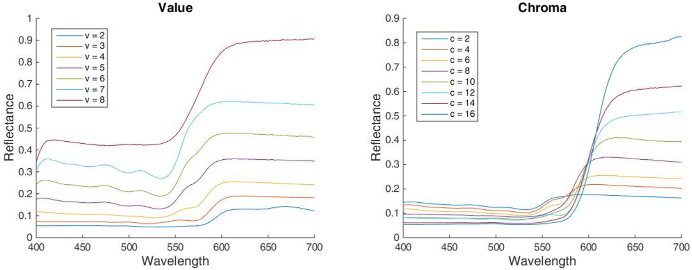
- Witzel et al. (2015) showed that the singularity index and chroma were correlated.



- We found correlations for chroma of 0.74, while for value correlation of 0.1.
- The hues 'green' and 'blue' allow smaller chroma, can explain why their singularity index smaller compared to 'red' and 'yellow'.

Properties of singularity index: value and chroma

- Why dependence on chroma and not value ?



- Value: at first approximation, it homogeneously deforms the reflectance curves via a multiplicative constant. Since the singularity index only takes into account ratios between elements of the Reflectance Matrices, a multiplicative constant has no influence on the singularity index.
- Chroma: deforms the reflectance functions non-homogeneously by enhancing the contrast in reflectance between wavelengths. Thus, it enlarges the contrast between reflection coefficients.

Von-Kries diagonal approach

- In an RGB camera, we have three sensors R, G and B, with small overlapping sensitivity functions. We denote by x the pixel position. Due to the disjoint property of the R,G,B sensors, we can approximately write:

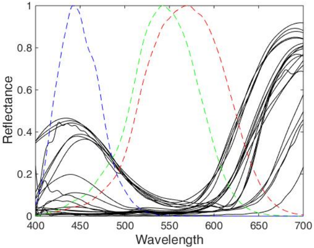
$$\begin{pmatrix} v_R^{\mathbb{S}}(x,E) \\ v_G^{\mathbb{S}}(x,E) \\ v_B^{\mathbb{S}}(x,E) \end{pmatrix} = \begin{pmatrix} S_R(x) & 0 & 0 \\ 0 & S_G(x) & 0 \\ 0 & 0 & S_B(x) \end{pmatrix} \begin{pmatrix} u_R(E) \\ u_G(E) \\ u_B(E) \end{pmatrix},$$

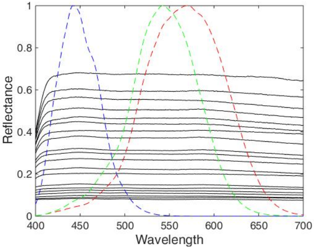
which, in turn, is equivalent to:

$$\begin{pmatrix} S_R(x) \\ S_G(x) \\ S_B(x) \end{pmatrix} = \begin{pmatrix} 1/u_R(E) & 0 & 0 \\ 0 & 1/u_G(E) & 0 \\ 0 & 0 & 1/u_B(E) \end{pmatrix} \begin{pmatrix} v_R^{\mathbb{S}}(x) \\ v_G^{\mathbb{S}}(x) \\ v_B^{\mathbb{S}}(x) \end{pmatrix}.$$

- Independent per component discount of the illuminant and extraction of reflection properties.

A closer look a the limits of II





- Lower accuracy when high peaks: in-homogeneous transformation over λ.
- Peak either around the beginning or the end of the visual spectrum.